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A maximum power point algorithm using the Lagrange method

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HIGHLIGHTS

► The Lagrange method is employed to find the maximum power point.
► A Seidel type iteration method is used to develop the algorithm.
► Uniqueness of the solution and convergence of the algorithm are mathematically shown.
► A Lagrange maximum power point tracker (LMPPT) is simulated using PSIM and MATLAB.

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ABSTRACT

The maximum power transfer problem is posed as a constrained optimization problem and the well-known Lagrange method is employed to address it in case of some simple circuits including a photovoltaic cell. In graphical terms, the maximum power point (MPP) is the point of tangency between the isopower contours and the source curve. For the example of a photovoltaic cell a Seidel type iteration is also suggested. A brief mathematical treatment of its convergence and uniqueness of the solution is given along with numerical simulations performed using MATLAB. This new algorithm lends itself to the implementation of the Lagrange MPP tracker (LMPPT). To show its working, the Solarex module MSX-60 is chosen and a simple buck-boost converter circuit is simulated with PSIM software. The terminal current, voltage and power are displayed showing the accuracy and the reliability of the proposed method under varying load and irradiance conditions.

1. Introduction

The maximum power transfer (MPT) theorem is one of the most important theorems in circuits and systems. In the case of a simple dc circuit, it is proved by differentiating the expression for load power, equating to zero and solving for the load resistance, which turns out to be equal to the source resistance. For an ac circuit involving impedances, the load impedance for maximum power equals the conjugate of the source impedance [1]. In both these two situations, the circuit is linear in nature. Of the total power delivered by a linear source, the maximum available to a linear load is 50%. MPT is kept in view wherever it is a concern; as such, circuits, transmission lines and antennas are designed with due consideration to impedance matching. In power systems engineering however, more than MPT, improving efficiency in power distribution is very important. But the growing energy crisis has spawned the pursuit of efficient power generation and delivery and a variety of devices and circuits, several of which are nonlinear, came into the fore. They include photovoltaic (PV) cells produced at industry and consumer scale, space-based solar energy systems, cord-less inductively coupled powering devices, implantable medical devices etc., and many other examples abound. Matching them for efficient energy extraction and tracking the maximum power point (MPP) has now become an intriguing research topic.

The interesting side of MPT is that, it is too simple in the case of a dc circuit but for a linear network of several ports or for a nonlinear circuit, it is indeed too hard. Zhong et al. reported an attempt to trace MPP by an extremum seeking controller in case of fuel cells [2]. Numerous papers appeared in the past two decades proposing ways to identify and track the MPP under varying load/source conditions. For example, [3] provides an interesting review of several algorithms known at the time to identify and track MPP from a PV generator. Exact analytical expression for the MPP in case of nonlinear circuits is known only for some selected cases. Rodriguez and Amaratunga [4] proposed an analytical method that captures the MPP in a small neighborhood.

Most of the MPP tracking schemes are iterative and rule-based in nature. In the recent past, there has been a flurry of activity

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toward the objective of developing fast algorithms to detect and track MPP and the associated hardware to realize them in real time [5–15]. Some of the MPP tracking (MPPT) methods that drew attention were (i) Perturb and Observe method (P&O) (ii) Constant Voltage method (CV), and (iii) Incremental Conductance (IncCond) method. Among these P&O was initially found to be fast but the IncCond suffered from low efficiency and slow convergence. The convergence of CV was reported to be fast but IncCond method was found to exhibit better stability [3,9]. Later, extensive improvements were made to the P&O method by optimizing the parameters of the algorithm. A parabolic prediction in the neighborhood of the MPP was described in Ref. [10] and a distributive MPPT method (DMPPT) was reported in Ref. [11]. The choices of the dc–dc converter topology and parameters, string size and inverter operating voltage have been found to be critical in the design steps of a PV system with DMPPT. An optimal design of a one-cycle controller of a single-stage inverter for PV applications was reported in Ref. [12]. Following this, Femia et al. [13] proposed an optimization technique for a double-stage grid-connected PV system. They have addressed the low-frequency drift problems arising across the bulk capacitor and came up with an analog control network that deals with the duty cycle of the boost converter involved in the controlling power electronics section. Bianconi et al. [14] devised a novel sliding mode control technique for MPP tracking that depends on the current drained by the shunt capacitor across the PV generator. The strength of this method lies in the fast detection of changes in irradiance levels and this new feature was found to be independent of PV array type. More recently, Petrone et al. [15] came up with what they called TEODI approach, which is a distributed scheme and essentially involves an analog circuitry to offset the low-frequency drifts.

In this article, the MPT problem is viewed, in general, as a constrained optimization problem and a graphical picture is presented. The load power expression $V_L I_L = P_L$ represents a family of hyperbolas. Also the load voltage V_L and the load current I_L in case of a simple dc circuit follow a straight line, and in other cases might bear an arbitrary nonlinear relation acting as a constraint. The MPT problem then becomes a constrained optimization problem in terms of the decision variables V_L and I_L to solve which, one could apply the familiar Lagrange method [16]. This approach was earlier adopted by Ahmadi et al. [17] in the context of very large scale photovoltaic power plants. In graphical terms, if the isopower contours and the source line are superimposed, the MPP is the point of tangency. We know that, in general, Lagrange method does not guarantee an optimal solution, but it is *fail-safe* in the sense that, if a solution is found, it must be true. Also the concavity of isopower contours assures a maximum at the point of tangency with the source curve. The overall difficulty of extremizing an objective function depends on the complexity of the source curve and may or may not be easier than the other methods, but the graphical picture offers helpful insights to the circuit designer.

As for the rest of the paper, in Section 2 we shall consider three simple examples: (i) a simple dc circuit, (ii) a diode circuit and (iii) a photovoltaic cell and proceed to find the MPP. The idea behind the first example is to illustrate the method, as standard textbooks such as [1] usually do not employ the Lagrange method. We give an exact expression for the MPP in the case of diode circuit. Thirdly, we address the photovoltaic cell and furnish an approach to obtain the MPP in three easy steps. Section 3 proposes a Seidel type iteration to solve the set of nonlinear equations arising from the Lagrange formulation. This section considers the important questions of existence, convergence and uniqueness that naturally ensue at the outset of such an iteration scheme. The conditions outlined and the possible convergence have been further explained by a simulation example in Section 4.

In order that the proposed method be of any practical utility, one has to devise how it can be implemented in practice. Section 5 details a general schematic for implementation. This configuration bears resemblance with the other existing MPPT systems. The paper subsequently ends in conclusions.

2. The Lagrange method to obtain maximum power

2.1. A simple dc circuit

Consider the simple dc circuit shown in Fig. 1(a) and the relation between V_L and I_L arising from Kirchhoff law shown in (b). In the introduction to diode and transistor circuits, a similar relation has been used and the line represented by it is the familiar load line. We shall call this the *source line* because it has the source resistance term R_S and describes the nature of the Thevenin circuit to the left of the load. In general this line describes how the source voltage E splits between R_S and R_L for a given quiescent current I_Q as indicated in Fig. 1(b). The power expression $V_L I_L = P_L$ may be graphically depicted in the V_L–I_L plane by the contour-plot shown in Fig. 2. The isopower contours are seen to be hyperbolas spreading away from the origin for greater value of power. If a hyperbolic contour corresponding to an arbitrarily small value of power is superimposed on the source line, it intersects at two points shown in Fig. 2 as α and β. But as the contour is gradually moved in search of higher power, α and β coalesce into a single point of tangency and that corresponds to the MPP. Beyond this, there is no intersection

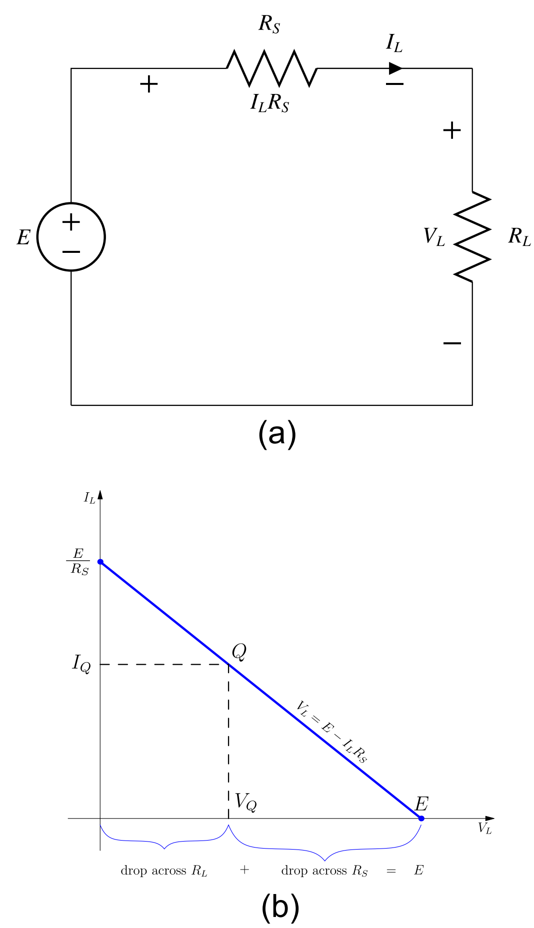

Fig. 1. (a) A simple dc circuit with a source voltage E resistance R_S and load R_L. (b) The relation between V_L and I_L called source line. For a given current I_Q this line shows how E is split between R_S and R_L.