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Bending of two-layer beams under uniformly distributed load – Analytical and numerical FEM studies

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A B S T R A C T

The subject of the paper are two-layer beams with various mechanical properties, thicknesses and widths of the layers. The original nonlinear hypothesis-theory of planar cross section is developed. Based on the principle of stationary potential energy three differential equations of equilibrium are obtained. The system of the equations is analytically solved and the deflections and normal and shear stresses of example beams are calculated. The analytical calculation results are compared with numerical solutions obtained with FEM (SolidWorks). These results are presented in Tables and Figures.

1. Introduction

Two-layer beams and plates are used as the parts of modern constructions. Gere and Timoshenko [1] presented a basic linear analytical model of a two-layer beam and described the distribution of normal stresses for example beams. Foraboschi [2] presented a nonlinear analytical model allowing to analyze the composite beams subject to transverse bending. The model takes into account the relative slip between the layers and predicts the stresses as well the load causing debonding of two-layer beams. The effect of various geometrical and material parameters was estimated through many exemplary variants of the beams. Ecsedi and Baksa [3] analyzed the static behaviour of two-layer beams with interlayer slip. The approach consisted in separate consideration of each layer using the Euler–Bernoulli hypothesis. The authors formulated a linear constitutive equation governing the relationship between the slip and the interlaminar shear force. A second order differential equation was derived the solution of which enabled to determine the slips and deflections. Le Grognec et al. [4] investigated buckling behavior of two-layer shear-deformable beams. The Timoshenko kinematic hypotheses were assumed for both layers, allowing to consider the relationship between the interface shear distribution and the corresponding slip. The proposed formulae were positively validated using the finite element calculations. Campi and Monetto [5] proposed a new approach to the analysis of two-layer beams with interlayer slip. The layers were considered as linearly elastic Timoshenko beams interconnected each with other. The problem was analytically solved for various boundary conditions and load cases, with a view to determine the interfacial forces causing debonding of the layers. Lenci and Rega [6] dealt with free vibration of a two-layer beam. The authors used the asymptotic development method in order to define the conditions allowing to ignore the axial and rotational inertia as well as shear deformations. Two variants of the model were assumed, permitting to compute the limit natural frequencies and their corrections. This enabled to determine the sensitivity of the beam to the adopted parameters. Yang et al. [7] presented a solution for a bilayer functionally graded cantilever beam subjected to concentrated load at its free end. The beam was modeled as a nonhomogeneous plane stress problem, with the elastic modulus of each layer varying in the thickness direction according to an arbitrary function. The method proposed by the authors could be easily upgraded for purposes of analysis of the functionally graded sandwich beams. He and Yang [8] analyzed two-layer composite beam subjected to seismic and moving loads, taking into account the beam higher order deformations. The authors developed a proper finite element using the principle of virtual work. The effects of the velocity of moving load, damping ratio, slenderness ratio and interfacial stiffness on the beam behaviour were studied. Numerical results obtained this way evidenced high accuracy of the dynamic analyses, better than in case of the classical and Reddy's models. Song et al. [9] proposed a model of two-layer smart composite Timoshenko beams, depicting the host composite beam and the layer composed of a piezoelectric transducer. The model was then evaluated by comparison with the results obtained from standard finite element analysis and, afterwards, it was applied with a view to investigate the waves guided in laminated composite beams. Monetto and Campi [10] improved a mathematical model, based on classical beam theories earlier proposed by the authors. The analytical solution based on the model enabled to achieve a generic configuration of the composite beam, characterized by various regimes coexisting along the interface. The approach

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allowed to simulate the course of the interface debonding of the layers, positively verified by the numerical analyses. Uddin et al. [11] formulated a nonlinear finite element model designed for accurate predicting of the behaviour of two-layered composite beams, with consideration of partial shear interaction. The one-dimensional finite element model developed for this purpose enabled to illustrate the geometric nonlinear effects of the beam. Effectiveness of the approach was assessed by comparison with previously published results and numerical results based on two-dimensional finite element modelling of the beam. Wen et al. [12] presented an analytical model of a two-layered composite beam, taking partially into account the shear effect. The model provided for the longitudinal displacements in both layers separately, in the direction normal to the beam axis. The governing equations were derived based on the principle of virtual work and then solved with the use of a Navier type solution technique. The results so obtained were compared to the previously published ones and to the results coming from numerical analyses. Časa et al. [13] developed a new mathematical model allowing to analyze the mechanical behaviour of two-layer composite beams, with consideration of interlayer slips between the layers. A cantilever 3D two-layer timber-concrete composite beam was then analyzed for various kinematic and equilibrium conditions. It was ascertained that the deformations in transverse and lateral planes of the beam are mutually independent. Hou and He [14] dealt with dynamic and static response of the two-layer partial interaction composite beams. The principle of virtual work allowed to develop a specialized finite element designed for the analysis. The numerical results obtained this way evidenced efficiency of the proposed method. It turned out significantly better in predicting the static response, natural frequencies and higher order free vibration beam modes than classic FEM method. Magnucki et al. [15] assumed a nonlinear hypothesis of planar cross-section deformation with a view to consider the behaviour of a beam with unsymmetrical variation of its mechanical properties. The Hamilton's principle allowed to derive two differential equations of motion. Solution of these equations provided the results illustrating the bending, buckling and free vibration of the beam. The additionally developed FEM model of the beam verified accurateness of the approach. Magnucki et al. [16] studied an unsymmetrical sandwich beam, assuming that the thicknesses and mechanical properties of the beam faces are different. The classical broken-line hypothesis served as a basis for formulation of the equations of motions. The values of deflection, critical force and natural frequency calculated for several variants of the beam were positively verified by FEM computation. Polus and Szumigala [17] presented the methods for calculating the bending resistance and the stiffness of aluminum – concrete composite beams. Two types of connection between the aluminum beam and the concrete slab were considered. The results so obtained were confirmed by laboratory tests.

The subject of the study are simply supported two-layer beams of length L, total depth h and different mechanical properties (E_1, ν_1, E_2, ν_2), thicknesses (h_1, h_2) and widths (b_1, b_2) of the layers (Fig. 1a). The beams are subjected to uniformly distributed load of intensity q (Fig. 1b). The bending problem of these beams with consideration of the shear effect is analytically and numerically FEM studied.

The objective of the study is formulation of analytical model of the two-layer beams (having different widths b_1 and b_2) with consideration of the original nonlinear hypothesis-theory of deformation of the planar cross sections, derivation of equilibrium equations and their solution. Both layers are made of different isotropic materials. The b_2/b_1 ratio varies from 1/6 (T-beam) to 1 (rectangular cross section). Moreover, the FEM numerical calculations and the comparative analysis of the results are carried out.

2. Analytical model of the two-layer beam

The classical Bernoulli-Euler beam theory is commonly used in solving the problems of beam bending. It entails the fact that a planar cross section of the beam remains planar after bending, of course in case of a small deflection. In consequence, the shear effect is ignored, but in long beams such an approach gives satisfactory results. However, the problems of short beams made of different layers require more sophisticated assumptions. Therefore, an individual hypothesis-theory of deformation of a planar cross section is formulated, making a basis for development of the analytical model of the two-layer beam. Any straight line perpendicular to the beam neutral axis takes the shape shown in Fig. 2 after bending.

The longitudinal displacements in particular layers, according to this hypothesis, are as follows:

- the upper layer-part ($-h_1 \leqslant y_1 \leqslant 0$)

$$u(x, y_1) = -(y_1 - y_0)\frac{dv}{dx} + f_d^{(1)}(y_1)u_1(x) - f_d^{(2)}(y_0)u_2(x), \tag{1}$$

where: the deformation function of the planar cross section of the upper layer-part

$$f_d^{(1)}(y_1) = \frac{1}{2 + 3\eta_{01}}\left[3(1 + 2\eta_{01}) + 3\eta_{01}\frac{y_1}{h_1} - \left(\frac{y_1}{h_1}\right)^2\right]\frac{y_1}{h_1}, \tag{2}$$

and

$f_d^{(2)}(y_0) = \frac{3 - 6\eta_{02} + 2\eta_{02}^2}{2 - 3\eta_{02}}\eta_{02}$ – value of the deformation function of the lower layer-part for $y_1 = y_0$, $\eta_{01} = y_0/h_1$, $\eta_{02} = y_0/h_2$ – dimensionless parameters, $0 \leqslant y_0$ – the position of the neutral axis (Fig. 2), $u_1(x)$, $u_2(x)$ – longitudinal displacements.

- the lower layer-part ($0 \leqslant y_1 \leqslant h_2$)

$$u(x, y_1) = -(y_1 - y_0)\frac{dv}{dx} + [f_d^{(2)}(y_1) - f_d^{(2)}(y_0)]u_2(x), \tag{3}$$

where: the deformation function of the planar cross section of the lower layer-part

$$f_d^{(2)}(y_1) = \frac{1}{2 - 3\eta_{02}}\left[3(1 - 2\eta_{02}) + 3\eta_{02}\frac{y_1}{h_2} - \left(\frac{y_1}{h_2}\right)^2\right]\frac{y_1}{h_2}. \tag{4}$$

Consequently, the strains and stresses in particular layers are as follows:

- the upper layer-part ($-h_1 \leqslant y_1 \leqslant 0$)

Strains

$$\varepsilon_x^{(1)}(x, y_1) = \frac{\partial u}{\partial x} = -(y_1 - y_0)\frac{d^2v}{dx^2} + f_d^{(1)}(y_1)\frac{du_1}{dx} - f_d^{(2)}(y_0)\frac{du_2}{dx}, \tag{5}$$

$$\gamma_{xy}^{(1)}(x, y_1) = \frac{dv}{dx} + \frac{\partial u}{\partial y_1} = \frac{df_d^{(1)}}{dy_1}u_1(x), \tag{6}$$

where: derivative of the deformation function

$$\frac{df_d^{(1)}}{dy_1} = \frac{3}{2 + 3\eta_{01}}\left[1 + 2\eta_{01} + 2\eta_{01}\frac{y_1}{h_1} - \left(\frac{y_1}{h_1}\right)^2\right]\frac{1}{h_1}.$$

Stresses

$$\sigma_x^{(1)}(x, y_1) = E_1\varepsilon_x^{(1)}(x, y_1),\ \tau_{xy}^{(1)}(x, y_1) = \frac{E_1}{2(1 + \nu_1)}\gamma_{xy}^{(1)}(x, y_1) \tag{7}$$

where: E_1, ν_1 – Young's modulus and Poisson ratio of the upper layer-part.

- the lower layer-part ($0 \leqslant y_1 \leqslant h_2$)

Strains

$$\varepsilon_x^{(2)}(x, y_1) = \frac{\partial u}{\partial x} = -(y_1 - y_0)\frac{d^2v}{dx^2} + [f_d^{(2)}(y_1) - f_d^{(2)}(y_0)]\frac{du_2}{dx}, \tag{8}$$

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