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Generalized viscoelastic model for laminated beams using hierarchical finite elements

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ABSTRACT

The formulation of an enriched hierarchical one-dimensional finite element suitable to analyze the rheological behavior of thick arbitrarily laminated beams is presented. The formulation is based on the Equivalent Single Layer (ESL) theory and was developed to allow the use of any high-order beam shear deformation theory (HBST) in a unified approach. A generalized Maxwell model was implemented to analyze the time-dependent behavior of the composite. The finite element employs local Lagrange and Hermitian support functions enriched with orthogonal Gram-Schmidt polynomials and is free of shear locking. The enriched macro elements can be used with very coarse meshes and the precision can be controlled without generating a new mesh. The formulation has been validated with numerical examples of symmetric and non-symmetric laminated beams using the three-dimensional finite element program PLCD.

1. Introduction

Sandwich structures are widely used as structural supporting elements due to their enhanced mechanical properties. These consist of two thin rigid layers that support much of the flexural load and a lightweight inner core, usually with energy dissipation capabilities. Consequently, these layered composite laminates are well suited for applications where vibration-induced stresses can produce problems such as fracture, fatigue, etc. These problems can be mitigated by the introduction of viscoelastic material core layers of different thickness [1]. The study of the vibration and dynamic response of viscoelastic structures and sandwich panels with viscoelastic cores has been studied over the years. There are several models based on layerwise theories for modelling of laminated structures with viscoelastic damping layers.

Moreira et al. [2] developed a layerwise model which can describe the high shear pattern developed inside a thin viscoelastic soft core. Later, they implemented a 4-node facet type quadrangular shell finite element, where the bending stiffness of the facet shell element is based on the Reissner–Mindlin assumptions and the plate theory is enriched with a shear locking protection [3]. Mahmoodi et al. [4] investigated the non-linear free vibrations of viscoelastic beams using the Kelvin-Voigt model and a multi-scale method was used to analytically raise the non-linear modal shapes and natural frequencies in beams. Araújo et al. [5] presented an optimal design and parameter estimation of frequency dependent passive damping of sandwich structures with viscoelastic core and a viscoelastic sandwich finite element model for the analysis of passive, active and hybrid structures [6,7], using a mixed approach, by considering a HSDT to represent the displacement field of the viscoelastic core, and a FSDT for the displacement fields of adjacent laminated face layers. The differential transformation method (DTM) in the frequency domain was employed by Arikoglu and Ozkol [1] to solve the motion equations of sandwich-composite beams including viscoelastic cores. Arvin et al. [8] developed a Finite Element Method (FEM) code to study the vibration frequencies of sandwich beams constituted by external layers of composite material and a viscoelastic core. Moita et al. [9] developed a finite element model for vibration analysis of active–passive damped multilayer sandwich plates, where the elastic layers are modelled using the classic plate theory and the core is modelled using the Reissener–Mindlin theory. They improved this formulation using Reddy's third-order shear deformation theory [10]. The finite element is obtained by assembly of N "elements" through the thickness, using specific assumptions on the displacement continuity at the interfaces between layers. Galupppi et al. [11] analytically solved the time-dependent problem of a laminated beam with an intermediate viscoelastic layer, whose response is modeled by a Prony-Series. Later, they applied the analysis to a structure composed of elastic glass layers joined by intermediate layers of viscoelastic polymers, under a history of loading and unloading [12]. Optimization of active and passive

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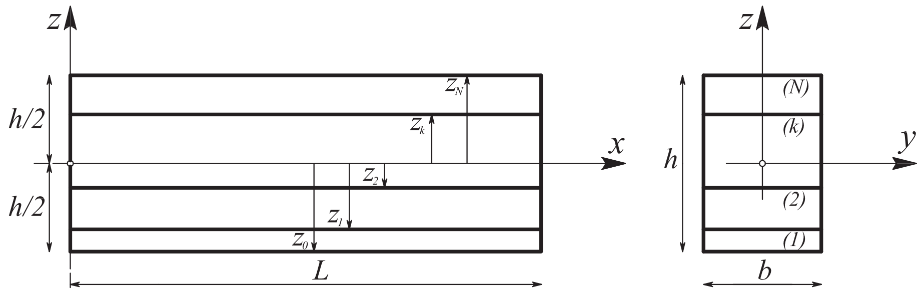

Fig. 1. Multilayered composite beam. Geometry and coordinate system.

damping using a new mixed layerwise finite element model was implement by Araújo et al. [13], where results are compared with an alternative optimization model, based on 3D finite elements using ABAQUS commercial package. Ferreira et al. [14] developed a layerwise finite element model using a unified formulation for the analysis of sandwich laminated plates with a viscoelastic core and laminated anisotropic face layers. Liu et al. [15] formulated a layerwise differential quadrature hierarchical finite element (DQHFE) model for the analysis of sandwich laminated plates with a viscoelastic core and laminated anisotropic face layers. The stiffness and mass matrices in these cases were obtained by Carrera's Unified Formulation (CUF) [16]. Lei et al. [17] established movement equations for Euler-Bernoulli beams using the Kelvin-Voigt model and the standard three-parameter viscoelastic model with velocity-dependent external damping. Li et al. [18] obtained closed form solutions for stresses and strains using the Laplace transform for simply-supported laminated functionally graded (FG) beams, considering viscoelastic interlayers. Dynamic behavior of sandwich composite beams using different shear deformation theories to formulate diverse layer-wise models (LW), implemented through FEM were studied by Loja et al. [19]. Kpeky et al. [20] proposed the modeling of sandwich structures with a soft core using a finite linear-hexahedral solid element, combining Automatic Differentiation (AD) with the Asymptotic Numerical Method (ANM). A multiobjective approach for optimization of passive damping for vibration reduction in sandwich structures is presented by Madeira et al. [21,22], for maximization of modal loss factors and minimization of weight of sandwich beams and plates with elastic laminated constraining layers and a viscoelastic core, using the Direct MultiSearch (DMS) solver. Nguyen et al. [23] implemented a triangular finite element that uses the Laplace transform for laminated viscoelastic composite plates based on an efficient higher order zigzag theory (EHOPT). Huang et al. [24] presented two integral finite elements, with compression and shear mechanisms of damping, to model sandwich type structures with soft core. Li et al. [25] developed a semi-analytical method to investigate the natural frequencies and modal shapes of a system of beams interconnected by a viscoelastic layer. The work of Latifi et al. [26] deals with geometrically non-linear transient analysis of sandwich beams with viscoelastic cores and composite laminated external layers. The non-linear dynamic instability of three-layer composite beams with a viscoelastic core subjected to combined lateral and axial loads was also studied by the same authors in Ref. [27]. Guo et al. [28] analyzed a sandwich beam where monolithic viscoelastic core was replaced by two periodically alternating viscoelastic ones to improve the flexural-wave attenuation performance. This work was later extended by Sheng et al. [29] to consider plate sandwich structures. Mustafa [30] studied laminated Timoshenko beams with two identical external layers bounded by a thin adhesive layer. Zhai et al. [31] analyzed the free vibration of five-layer sandwich plates with two viscoelastic cores using the first order shear deformation theory (FSDT); the motion equations were derived using Hamilton's principle and solved by the closed-form Navier method. It should be noted that he inner-core damping layers undergo strong shear strains, due to the relative motion of the layers. For this reason, it is important to use an appropriate kinematics that considers the effect of the shearing through advanced structural theories [32]. Asik and Tezcan [33], Bennison and Davies [34] and Ivanov [35], among others, showed that an appropriate consideration of the viscoelastic interlayer shear behavior is essential for an accurate modeling allowing an efficient design.

The formulation of an enriched hierarchical one-dimensional finite macro-element suitable to analyze the rheological behavior of thick arbitrarily laminated beams, including soft-core sandwich ones, is presented. The formulation is based on the Equivalent Single Layer (ESL) theory which is well suited in design stages or optimization processes where repetitive computations and a good balance between accuracy and resolution speed are required. The formulation is based on the Equivalent Single Layer (ESL) theory and was developed to allow the use of any high-order beam shear deformation theory (HBST) in a unified approach. A generalized Maxwell model was implemented to analyze the time-dependent behavior of the visco-elastic layers. The finite element employs local Lagrange and Hermitian support functions enriched with orthogonal Gram-Schmidt polynomials. The obtained finite element is free of shear locking and thin beams can be studied with the same formulation without resorting to the use of reduced integration [32,36]. The formulation has been validated with numerical examples of symmetric and non-symmetric laminated beams using the three-dimensional finite element program PLCD.

2. Formulation of the mechanical problem

2.1. Kinematics of deformation

A laminated beam of length L, width b and total thickness h, as shown in Fig. 1 is considered. An orthogonal Cartesian axis system (x, y, z) is used, with the axis x oriented along the longitudinal axis of the beam, the plane $x - y$ coincides with the middle plane and the z axis is perpendicular to the middle plane, resulting in a three dimensional domain where $0 \leqslant x \leqslant L, - b/2 \leqslant y \leqslant b/2, - h/2 \leqslant z \leqslant h/2$. The laminated beam is composed of N layers of different elastic and viscoelastic materials. The index k denotes the layer number from the bottom to the top of the beam, therefore the layer $k - th$ lies between $z_{k-1} \leqslant z \leqslant z_k$ and its thickness is $h^{(k)}$. The kinematics of the laminated beam is characterized by the displacements of its midline and occurs in the plane $x - z$. The components of the displacement field are obtained using different shear deformation theories in the frame of ESL theories, adopting the following general form:

$$\begin{aligned} u_1(x, z, t) &= u(x, t) - z\frac{\partial w(x, t)}{\partial x} + f_i(z)\phi(x, t) \\ u_2(x, z, t) &= 0 \\ u_3(x, z, t) &= w(x, t) \end{aligned} \tag{1}$$

where u_1, u_2, u_3 are the displacements of any material point in the beam domain along the axes x, y, z, respectively; u, w are the longitudinal (x axis) and transverse (z axis) displacements of generic points located on the beam longitudinal axis; ϕ is the additional rotation of the normal to the midplane; $f_i(z)$, $i = 1, ...,n$ represents the shape function that determine the distribution of strains in the beam thickness for different