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Best non-polynomial shear deformation theories for cross-ply single skin and sandwich shells

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ABSTRACT

This paper presents Best Theory Diagrams (BTDs) constructed from non-polynomial terms to identify best shell theories for bending analysis of cross-ply single skin and sandwich shell panels. This structure presents a constant radii of curvature. The shell theories are constructed using Axiomatic/Asymptotic Method (AAM). The different shell theories are described using the Carrera's Unified Formulation. The governing equations are derived from the Principle of Virtual Displacement (PVD). Navier-Type closed form solution is used for solving the bending problem of simply supported doubly curved shell panels subjected to bi-sinusoidal transverse pressure. The BTDs built from non-polynomial functions are compared with Maclaurin expansions. Spherical shell panels with different layer-configurations are investigated. The results demonstrated that the shell models obtained from the BTD using non-polynomial terms can improve the accuracy obtained from Maclaurin expansion for a given number of unknown variables of a displacement field.

1. Introduction

Shells have increased structural stiffness compared to plates. Shells have excellent load capability due to its curvature and it is attractive shape geometry in the industry. On the other hand, fiber composite [1] provide significant improvements in property strength and stiffness over conventional metal alloys. Goraj [2] divided the advantages of composite over classical metals in four characteristics: weight reduction, reduced cost. Those may be the reasons why composite shell structures are widely used in many industries, such as, aerospace, automobile, mechanical, civil and marine.

The abovementioned industries and others need simplified models for calculating stresses and displacement during the design process, manufacturing, monitoring the performance of the structure in operation, and for decision making in any other kind of circumstances. 3D elasticity solutions need a large amount of degree of freedom and high computational cost. In order to reduce the computational cost, many researchers developed 2D theories for modeling the stresses and displacements of shell structures over the last seventy years. The first approach was given by Love [3] who extended the Kirchhoff hypothesis valid for thick plates to shell structures. The work by Love was the foundation for the classical shell theory (CST), this theory was developed in several classical textbooks [4–7]. The CST neglected the transverse shear and transverse normal stresses and this issue generates inaccuracies for thin shells. The introduction of transverse shear stress was introduced by Hildebrand et al. [8] who proposed the first order shear deformation theory (FSDT). The limitation of this theory is related to the necessity of a shear correction factor [9]. Higher order shear deformation theories (HSDT) were stablished to reduce the inaccuracies of FSDT and CST. The displacement field of HSDT normally is based on the quadratic, cubic and higher order expansions [10–12] or non-polynomial robust expansions [13–15] along the thickness direction. A generalized way to write HSDT can be achieved by using a Unified Formulation (UF) [16–17]. HSDT and UF can be formulated as: Equivalent single layer (ESL) and layer-wise (LW) theories [18–19]. The ESL approach considered a multilayered shell as a single lamina. The LW approach each lamina is considered separately, therefore, the displacements and stresses distribution present quasi-3D capabilities. However, the computational cost of LW is high in comparison with ESL models.

The refined models presented in this paper follow the strategy for compactness introduced by Carrera's Unified Formulation (CUF) [20]. The governing equations are written in term of a few fundamental nuclei which do not formally depend on the order of expansion used in the thickness direction and on the description of the expansion variable. CUF can be used for solving acoustic problems [21], piezo-electric [22]

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and dynamical rotating structures [23]. Recently, several researchers [24–27] introduce non-polynomial functions along CUF to build new plate models. In the author's best knowledge, this is one of the first paper that introduces a unified theory for shells structures with non-polynomial expansion. This paper used the Navier closed form solution to solve the highly couple differential equations that govern a cross-ply laminated composite shells.

Axiomatic/asymptotic method (AAM) was developed by Carrera and Petrolo [28]. This strategy allows recognizing the effectiveness of each displacement variables of an arbitrary refined shell theory. The AAM method was applied for solving functionally graded structures [29], isotropic and laminated shell [55], and piezo-electrical problems [22,30]. The graphical representation of AAM method is presented using the Best theory diagram (BTD) [30] which, for a given problem, the computational cheapest mathematical model for a given accuracy can be read. Yarasca and co-authors [31,57] presented an effective N-objective optimization method using genetic algorithm considering displacement and stresses as output parameters. BTD were given considering Maclaurin, high order zig-zag, trigonometrical, exponential and hyperbolic expansion terms in order to study the importance of such expansion functions in the structural response. The authors found remarkable BTDs.

Tornabene et al. [32] used the differential quadrature technique and the Newton-Raphson iteration to obtain the solution of static problems related with laminated composite plate and shells resting on nonlinear elastic foundation. Viola et al. [33] presented a generalized nine-parameter HSDT for studying the static response for different doubly-curved laminated composite shell and panels. Wang et al. [34] presented linear vibration analysis of functionally graded carbon nanotube reinforced composite doubly-curved panels and shells of revolution with arbitrary boundary conditions using a novel semi-analytical method. Brischetto [35] proposed an exact 3D solution for the study of plates and shells. The solution is based on layer-wise approach and the second order differential equations are solved using a redouble of variable and the exponential matrix method. D'Ottavio [48] used the Sublaminate Generalized Unified Formulation which main idea is to group the plies into a number of smaller units called sublimated, each of them is characterized by an independent, variable-kinematic theory for bending of sandwich plates.

In this paper, a unified formulation is used to evaluate different hybrid reduced shell theories for laminated and sandwich shells. Consequently, BTDs are obtained using the average of error of the calculated error of three different displacements (u_α, u_β, u_z) and six different stresses ($\sigma_{\alpha\alpha}$, $\sigma_{\beta\beta}$, $\tau_{\alpha\beta}$, $\tau_{\alpha z}$, $\tau_{\beta z}$, σ_{zz}) by the multi-point criteria for stresses and displacements used by Carrera et al. [55]. BTDs are calculated for each non-polynomial model and are compared with Maclaurin expansion BTDs.

The paper is organized as follows: Section 2 describes the compact unified non-polynomial shear deformation formulation for shells; the governing equations and closed form solutions are presented in Section 3. The procedure for stress recovery for transverse shear and normal stress is outlined in Section 4. Axiomatic/asymptotic method is described in Section 5. The results and discussions are presented in Section 6. Finally, the conclusions are given in Section 7.

2. Compact non-polynomial shear deformation for shells

The coordinate system and the graphical representation of a multilayered spherical shell is presented in Fig. 1. The integer *"k"* denotes the layer number. The in-plane coordinates are denoted as "α" and "β", while the thickness coordinate is represented as "z". The radii of curvatures along the mid-surface domain are "R_α" and "R_β".

In the framework of a compact and unified formulation [20], the shell displacement can be described as:

$$\delta u_{(\alpha,\beta,z)} = F_\tau(z)\delta u_\tau(\alpha, \beta);$$

$$u_{(\alpha,\beta,z)} = F_s(z)u_s(\alpha, \beta);\ \tau,\ s = 0,\ 1,\ 2,\ 3,\ \cdots, N \tag{1}$$

where u is the displacement vector denoted as (u_α, u_β, u_z) whose displacements are along α, β and z reference axes and F_τ and F_s are the shear strain shape functions. The displacement variables is denoted as u_s and its variation as δu_τ. The order of expansion is represented as N.

In ESL scheme, the multilayer shell is considered as a single equivalent lamina. The ESL theory is denoted as EDN. If Maclaurin expansion is used, ED4 is:

$$\begin{aligned} u_\alpha &= u_{\alpha 0} + z u_{\alpha 1} + z^2 u_{\alpha 2} + z^3 u_{\alpha 3} + z^4 u_{\alpha 4} \\ u_\beta &= u_{\beta 0} + z u_{\beta 1} + z^2 u_{\beta 2} + z^3 u_{\beta 3} + z^4 u_{\beta 4} \\ u_z &= u_{z0} + z u_{z1} + z^2 u_{z2} + z^3 u_{z3} + z^4 u_{z4} \end{aligned} \tag{2}$$

In addition to the classical Maclaurin polynomial expansion, this paper uses three different non-polynomial fields: trigonometrical expansion (trig.), hyperbolic expansion (hyp.), and exponential expansion (exp.), see Table 1. The principal aim of this work is to evaluate several shear strain shape functions proposed in the literature and use them for creating different Best Theory Diagrams for shells. Table 2 presents several shear strain shape functions used in this publication.

The displacement field can be also written in a Layer-wise approach (LW), the main feature is that in LW scheme each layer of the shell structure is modelled separately as follows:

$$\begin{aligned} u &= F_s(z)u_s(\alpha, \beta) = F_t(\zeta_k)u_t^k + F_b(\zeta_k)u_b^k + F_r(\zeta_k)u_r^k,\ s \\ &= t,\ b,\ r;\ r = 2,\ 3,\ \cdots, N \end{aligned} \tag{3}$$

where *"b"* and *"t"* denoted the bottom and the top of the shell panel. The thickness functions are denoted as F_s and are defined by the use of Legendre's polynomials $P_j = P_j(\zeta_k)$ of j order. It is recalled that the LW theories are written in local dimensionless coordinates $\zeta_k = \frac{2z^k}{h^k}$ and are presented next:

$$\begin{aligned} &P_0 = 1,\ P_1 = \zeta_k,\ P_2 = \frac{3\zeta_k{}^2 - 1}{2},\ P_3 = \frac{5\zeta_k{}^3 - 3\zeta_k}{2},\ P_4 = \frac{35\zeta_k{}^4 - 30\zeta_k{}^2 + 3}{8} \\ &F_t = \frac{P_o + P_1}{2},\ F_b = \frac{P_o - P_1}{2},\ F_r = P_r - P_{r-2},\ r = 2,\ 3, \cdots N \end{aligned} \tag{4}$$

The acronym for this theory is denoted as LDN, where L denoted Layer-wise approach, D for the Principle of Virtual Displacements, and N is the order of expansion.

3. Governing equations and closed form solution

The stress (σ^k) and strain (ε^k) are grouped as follows:

$$\begin{aligned} &\sigma_p^k = [\sigma_{\alpha\alpha}^k \sigma_{\beta\beta}^k \tau_{\alpha\beta}^k]^T,\ \sigma_n^k = [\tau_{\alpha z}^k \tau_{\beta z}^k \sigma_{zz}^k]^T, \\ &\varepsilon_p^k = [\varepsilon_{\alpha\alpha}^k \varepsilon_{\beta\beta}^k \gamma_{\alpha\beta}^k]^T,\ \varepsilon_n^k = [\gamma_{\alpha z}^k \gamma_{\beta z}^k \varepsilon_{zz}^k]^T, \end{aligned} \tag{5a-d}$$

The subscript *"p"* denotes in-plane component and *"n"* refers to the out-plane component. The strain linear relations are described as follow:

$$\varepsilon_p^k = (D_p + A_p)u^k,\ \varepsilon_n^k = D_n u^k = (D_{np} + D_{nz} - A_n)u^k,$$

$$D_p{}^k = \begin{bmatrix} \frac{1}{H_\alpha{}^k}\frac{\partial}{\partial\alpha} & 0 & 0 \\ 0 & \frac{1}{H_\beta{}^k}\frac{\partial}{\partial\beta} & 0 \\ \frac{1}{H_\beta{}^k}\frac{\partial}{\partial\beta} & \frac{1}{H_\alpha{}^k}\frac{\partial}{\partial\alpha} & 0 \end{bmatrix},\ D_{np}{}^k = \begin{bmatrix} 0 & 0 & \frac{1}{H_\alpha{}^k}\frac{\partial}{\partial\alpha} \\ 0 & 0 & \frac{1}{H_\beta{}^k}\frac{\partial}{\partial\beta} \\ 0 & 0 & 0 \end{bmatrix},\ D_{nz}{}^k = \begin{bmatrix} \frac{\partial}{\partial z} & 0 & 0 \\ 0 & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \end{bmatrix},$$

$$A_p{}^k = \begin{bmatrix} 0 & 0 & \frac{1}{H_\alpha{}^k R_\alpha{}^k} \\ 0 & 0 & \frac{1}{H_\beta{}^k R_\beta{}^k} \\ 0 & 0 & 0 \end{bmatrix},\ A_n{}^k = \begin{bmatrix} \frac{1}{H_\alpha{}^k R_\alpha{}^k} & 0 & 0 \\ 0 & \frac{1}{H_\beta{}^k R_\beta{}^k} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{6a-g}$$

where "H_α" and "H_β" are denoted as the metric coefficients and are