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A simplified lumped damage model for reinforced concrete beams under impact loads

Daniel V.C. Teles, Mateus C. Oliveira, David L.N.F. Amorim*

Laboratory of Mathematical Modelling in Civil Engineering, Post-graduation Programme in Civil Engineering, Federal University of Sergipe, São Cristóvão, Brazil

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ABSTRACT

This paper presents a simple lumped damage model to analyse reinforced concrete beams under impact loads. The proposed model is based on the thermodynamics of irreversible processes. In this framework, key concepts of fracture and damage mechanics are applied on plastic hinges. Since the inelastic phenomena are modelled by two internal variables, named damage and plastic rotation, the plastic hinges are now called inelastic hinges. Therefore, the proposed finite element is an assemblage of an elastic beam with two inelastic hinges at its edges. Experimental results are used to propose the evolution laws of the internal variables. The proposed model is applied to other experiments and the obtained results show its accuracy. Finally, if an extensive experimental campaign could be carried out, a practical engineering criterion can be elaborated in terms of the damage variable (concrete cracking), which may be useful as a decision-making criteria for retrofit worthiness.

1. Introduction

Structural accidents are a global issue and, in many cases, result in fatal victims. The genesis of structural accidents is connected to, at least, one construction activity, such as structural design, building process and using. In this context, human actions must be taken into account, such as the lack of technical capacity, the use of inappropriate materials, the material deterioration due to aging and severe external actions.

Although an extensive approach on this theme is possible, this paper focuses on severe external actions in reinforced concrete structures, specifically impact loadings. Severe external actions are diverse in cause and effect. For instance, a gas leak may cause an explosion that damage structural elements, a heavy vehicle could collide with structures or a terrorist strike may occur on buildings with many people. Considering only impact loadings, the applications can be for civil or military purposes. The main idea is to consider an analysis that may predict structural response due to different impact loadings, such as various types of accidents and aggressive strikes.

Engineers and researchers discuss this theme around the world. Regarding impact loadings, several experimental studies were carried out in order to analyse the structural behaviour [1–15]. Some of those studies also conduct numerical analyses, using finite elements and at least one classic nonlinear theory. For instance [13], presented a simplified finite element description including a continuum damage variable that includes strain rate effects modelled to treat dynamic loading at low, medium and high impact velocities.

During the last decades, nonlinear theories were proposed in order to achieve an accurate description of the structural behaviour. Among classic nonlinear theories, three options are emphasised: theory of plasticity, fracture mechanics and damage mechanics.

Theory of plasticity is probably the most known nonlinear framework. In this theory, local plastic strains are responsible to account for the deterioration process.

Classic fracture mechanics is a powerful tool to analyse propagation of a small number of cracks in continuum media. Differently from plasticity, in fracture mechanics a small number of discrete cracks characterises the deterioration process. The crack propagation is taken into account by energy principles. This approach shows remarkable success for structures with simple geometries, few cracks and homogeneous materials.

Classic damage mechanics is the most recent great theory to describe the processes of deterioration and structural failure. The fundamental idea is quite simple: the introduction of an internal variable that characterises the material deterioration. This variable, called damage, takes values between zero and one. Damage is introduced in the behaviour laws also using simple concepts: an effective stress combined to a strain equivalence hypothesis. This theory has been successful in the description of several deterioration mechanisms in continuum media.

However, despite their accuracy and importance, the aforementioned theories (theory of plasticity, fracture and damage mechanics) present drawbacks or limitations in terms of computational costs or

* Corresponding author.
E-mail address: david.amorim@ufs.br (D.L.N.F. Amorim).

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structural modelling that difficult their use for several engineering applications. It is not only because these models use tens of thousands of finite elements, but the main issue is that data input and transformation of numerical results in engineering decisions are essentially human activities. Note that, nowadays, computer costs are becoming a less important factor, however human effort is certainly not.

Alternatively, a recent nonlinear theory, called lumped damage mechanics (LDM), was introduced in the early nineties in order to achieve more realistic results of frame structures. LDM applies some key concepts from fracture [16] and damage mechanics [17] in plastic hinges, such as the Griffith criterion and the strain equivalence hypothesis. Therefore, LDM models usually present suitable results for practical problems.

LDM was initially developed and formally presented by [18–20] to analyse reinforced concrete frames under seismic loads. Such models [18–20] used the generalised Griffith criterion and the strain equivalence hypothesis to quantify concrete cracking and reinforcement yielding. Afterwards, the LDM framework was extended to problems involving RC buildings and bridges [21–28], steel frames [29–32], RC arches [33], tunnel linings and unreinforced masonry arches [34]. Regarding LDM models for steel frames, [29] proposed that the local buckling can be estimated by a damage variable, [30–31] developed a formulation that quantifies steel cracking due to low-cycle fatigue and [32] presented a reliability analysis of steel frames using LDM. An arched finite element was developed to analyse RC structures by [33], and [34] proposed a new LDM model for plain concrete. On the other hand, the main limitations of the models are that the parameters of steel frames subjected to local buckling must be determined only by experiments and the plain concrete model presents an internal parameter which physical meaning was not entirely understood.

Despite of several lumped damage models so far proposed, none of them analyse reinforced concrete structures under severe external actions, as impact loadings. In the light of the foregoing, this paper presents a new lumped damage model to analyse reinforced concrete beams under impact loads. In the proposed approach, the impact aftermath is modelled by a thermodynamic formulation, where the concrete cracking is accounted for a damage variable and the reinforcement yielding is quantified by plastic deformations. Such damage variable quantifies the cracking density, which characterises the stiffness reduction of the structural member. In this sense, the damage variable translates the cracking density in a scale from zero to one. Therefore, the estimate of damage in a reinforced concrete beam under impact loads might be useful as a practical engineering criterion as well as for damage identification purposes in structural health monitoring (see e.g. [35–36]).

This paper is organised as follows. Section 2 presents the proposed lumped damage model and its application for impact loadings. Section 3 depicts an example that illustrates the practical application of the proposed model. A simple flowchart for practical applications is also presented in Section 3.

2. Proposed simplified lumped damage model for RC beams under impact loading

2.1. Thermodynamic approach

A lumped damage beam is an assemblage of an elastic beam with two inelastic hinges at its edges (Fig. 1a). The deformed shape of the beam can be described by the generalised deformations ϕ_i and ϕ_j, which are two flexural rotations (Fig. 1b). Therefore, the generalised deformation matrix is given by:

$$\{\mathbf{\Phi}\} = \{\phi_i \quad \phi_j\}^T \tag{1}$$

where the superscript T means 'transpose of'.

Now the generalised stress matrix $\{\mathbf{M}\}$ is defined as a set of bending moments, M_i and M_j (Fig. 1c), that are conjugated to the generalised deformations.

$$\{\mathbf{M}\} = \{M_i \quad M_j\}^T \tag{2}$$

Assuming that the deformation equivalence hypothesis is valid [37], the generalised deformations matrix can be expressed by:

$$\{\mathbf{\Phi}\} = \{\mathbf{\Phi^e}\} + \{\mathbf{\Phi^d}\} + \{\mathbf{\Phi^p}\} \tag{3}$$

being $\{\mathbf{\Phi^e}\}$ the elastic generalised deformation matrix, $\{\mathbf{\Phi^d}\}$ the matrix that accounts for concrete cracking by means of damage variables at each hinge (d_i and d_j), and $\{\mathbf{\Phi^p}\}$ the portion that quantifies the yielding of the longitudinal reinforcement (Fig. 1d).

Note that the damage variables take values between zero and one. If damage is null, there is no cracks in concrete; but if damage tends to one, the cracks surface is near to reach cross section area, breaking the beam in two parts.

Considering that the beam presents a certain level of damage and plastic deformation, the Helmholtz specific free energy [38–39] can be denoted by:

$$\chi = \frac{1}{2}\{\mathbf{\Phi} - \mathbf{\Phi^p}\}^T[\mathbf{F}(\mathbf{D})]^{-1}\{\mathbf{\Phi} - \mathbf{\Phi^p}\} \tag{4}$$

where χ is the total free energy potential and $[\mathbf{F}(\mathbf{D})]$ is the beam flexibility matrix (see Appendix A), given by:

$$[\mathbf{F}(\mathbf{D})] = \begin{bmatrix} \frac{L}{3EI(1-d_i)} & -\frac{L}{6EI} \\ -\frac{L}{6EI} & \frac{L}{3EI(1-d_j)} \end{bmatrix} \tag{5}$$

being $\{\mathbf{D}\} = \{d_i \quad d_j\}$ the set of damage variables at the hinges, L the length of the beam (Fig. 1b), E the Young modulus and I the inertia moment.

In order to ensure that the proposed model is thermodynamically admissible, the Clausius-Duhem inequality must be complied i.e. the dissipation of the deformation process cannot be negative. By assuming an isothermal process and the case of classic beam theory, such inequality is written as:

$$\{\mathbf{M}\}^T\{\dot{\mathbf{\Phi}}\} - \dot{\chi} \geqslant 0 \tag{6}$$

where the dot above the variable means temporal differentiation.

The second part of the Clausius-Duhem inequality can be expressed as:

$$\dot{\chi} = \left\{\frac{\partial\chi}{\partial\{\mathbf{\Phi}\}}\right\}^T\{\dot{\mathbf{\Phi}}\} + \left\{\frac{\partial\chi}{\partial\{\mathbf{\Phi^p}\}}\right\}^T\{\dot{\mathbf{\Phi}}^\mathbf{p}\} + \left\{\frac{\partial\chi}{\partial\{\mathbf{D}\}}\right\}^T\{\dot{\mathbf{D}}\} \tag{7}$$

by assuming that the thermodynamic potential can be linearized in the neighbourhood of the current values of the state variables.

Then, the Clausius-Duhem inequality is rewritten by substituting (7) in (6):

$$\left(\{\mathbf{M}\}^T - \left\{\frac{\partial\chi}{\partial\{\mathbf{\Phi}\}}\right\}^T\right)\{\dot{\mathbf{\Phi}}\} - \left\{\frac{\partial\chi}{\partial\{\mathbf{\Phi^p}\}}\right\}^T\{\dot{\mathbf{\Phi}}^\mathbf{p}\} - \left\{\frac{\partial\chi}{\partial\{\mathbf{D}\}}\right\}^T\{\dot{\mathbf{D}}\} \geqslant 0 \tag{8}$$

The derivative of the thermodynamic potential with respect to the state variables is given by the following expressions:

$$\left\{\frac{\partial\chi}{\partial\{\mathbf{\Phi}\}}\right\} = [\mathbf{F}(\mathbf{D})]^{-1}\{\mathbf{\Phi} - \mathbf{\Phi^p}\} = \{\mathbf{M}\} \tag{9}$$

$$\left\{\frac{\partial\chi}{\partial\{\mathbf{\Phi^p}\}}\right\} = -[\mathbf{F}(\mathbf{D})]^{-1}\{\mathbf{\Phi} - \mathbf{\Phi^p}\} = -\{\mathbf{M}\} \tag{10}$$

$$\left\{\frac{\partial\chi}{\partial\{\mathbf{D}\}}\right\} = \begin{Bmatrix} -\frac{M_i^2 L}{6EI(1-d_i)^2} \\ -\frac{M_j^2 L}{6EI(1-d_j)^2} \end{Bmatrix} = \begin{Bmatrix} -G_i \\ -G_j \end{Bmatrix} = \{\mathbf{Y}\} \tag{11}$$

where $\{\mathbf{M}\}$, $-\{\mathbf{M}\}$ and $\{\mathbf{Y}\} = \{-G_i \quad -G_j\}^T$ are the thermodynamic variables associated to $\{\mathbf{\Phi}\}$, $\{\mathbf{\Phi^p}\}$ and $\{\mathbf{D}\}$, respectively, being G_i and G_j the damage driving moments of the hinges [37]. Note that Eqs. (9) and