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Nonlinear wave scattering at the interface of granular dimer chains and an elastically supported membrane

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ABSTRACT

In this work we study computationally nonlinear wave scattering at the flexible interfaces of 1D dimer granular chains with a square membrane with a linear uniform elastic foundation. A computational algorithm that combines successive iterations and interpolations is developed to accurately model the highly nonlinear and non-smooth scattering of impeding pulses from the granular chains to the flexible boundary. Energy localization (through the excitation of local transient breathers), intense wave transmission or reflection, as well as strong pulse scattering in the frequency/wavenumber domain are detected for varying mass ratios of the dimers and stiffness of the elastic foundation of the membrane. Moreover, it is found that the realization of resonances or anti-resonances in the dimer granular chains at different mass ratios has significant effects on the nonlinear wave scattering at the flexible boundary. Interestingly enough, an inverse relation between the foundation stiffness and the residual energy transferred to the membrane from the impulsively excited dimer is found. Finally, we show that the energy exchanges between two granular chains interacting through the flexible foundation strongly depend on the distance between them. The presented results and the associated computational method discussed in this work contribute to the predictive modeling and design of granular media with flexible interfaces.

1. Introduction

Wave propagation in granular media composed of ordered or disordered arrays of discrete particles (granules) has attracted considerable attention from both theoretical and practical points of view. Due to their highly tunable and tailorable acoustic properties, granular media have been considered for various potential applications, such as nonlinear acoustic lenses (Spadoni and Daraio, 2010; Donahue et al., 2014), shock and energy absorbing layers (Melo et al., 2006) and passive acoustic filters (Lawney and Luding, 2014).

The pioneering research in the field of wave propagation in one-dimensional (1D) ordered granular media was carried out by Lazaridi and Nesterenko (1985), Nesterenko (1983), who discovered analytically, in numerical simulations and in experiments a new type of spatially localized, coherent, and shape preserving strongly nonlinear solitary wave (Nesterenko, 2001). The speed of these strongly nonlinear solitary waves has a stronger dependence on their amplitude than for weakly nonlinear Krteweg-de Vires (KdV) type solitary waves, which renders the former passively tunable with respect to energy (Nesterenko, 2001). Based on the earlier works by Coste et al. (1997) performed several experiments and observed a good agreement between experimental measurements and theoretical predictions. It is worth mentioning that the acoustics of 1D homogeneous granular chains (i.e., composed of a number of identical spherical elastic granules) depend heavily on the ratio of the amplitude of the wave and the pre-compression force between granules (Nesterenko, 2018). Indeed, in the case of no pre-compression, the essentially nonlinear inter-particle Hertzian interactions and possible separations along with ensuing collisions between adjacent granules yields complete absence of linear acoustics and zero speed of sound compression waves (as defined in classical acoustics), resulting in a “sonic vacuum” (Nesterenko, 2001; Nesterenko, 1994). In this case Nesterenko solitary waves are the main disturbances propagating in the sonic vacuum, and are characterized by strong dependence of their speed on amplitude, instead of the sound waves (phonons) that are the main disturbances in classical linear wave dynamics with speed independent of amplitude (Nesterenko, 2001). Moreover, the class of granular or non-granular nonlinear “sonic vacua” possesses highly complex nonlinear dynamics and acoustics, such as propagating solitary pulses (Jayaprakash et al., 2011a), travel-

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ing waves (Starosvetsky and Vakakis, 2010), a mixture of solitary and nonlinear shear waves (Zhang et al., 2015), frequency bands or breathers (Hasan et al., 2015), extreme nonlinear energy exchanges and "energy explosions" (Zhang et al., 2018), and strongly non-reciprocal acoustics (Zhang et al., 2016). Conversely, in the regime of strong pre-compression between granules (wave amplitude is very small in comparison with precompression stress), the dynamics and acoustics of granular media become weakly nonlinear, linearizable (i.e., a speed of sound in the sense of classical acoustics can be defined) and smooth (Nesterenko, 2001; Lazaridi and Nesterenko, 1985). It follows that depending on the ratio of the wave amplitude to the precompression stress, the dynamics and acoustics of 1D granular chains can be highly tunable, ranging from strongly nonlinear and non-smooth, to weakly nonlinear and smooth (Nesterenko, 2018). Extensions to higher dimensional ordered granular media were reviewed in Starosvetsky et al. (2017), Tiwari et al. 2017).

In addition to 1D homogeneous chains, the acoustics of 1D diatomic (dimer) granular chains have also been studied (Jayaprakash et al., 2011b, 2012a,b, 2013a,b; Herbold et al., 2009; Kim et al., 2015; Potekin et al., 2013; Rosas and Lindenberg, 2017), i.e., of ordered granular chains consisting of repetitive pairs of "light" and "heavy" granules. Theoretical analysis and experimental studies (Kim et al., 2015; Potekin et al., 2013) revealed that these non-dissipative chains with different mass ratio can support countable infinities of resonances and anti-resonances, yielding to either drastic attenuation of propagating pulses, or propagation of undistorted propagating solitary pulses, respectively.

Typically, the dynamics and acoustics of 1D granular chains are studied using free or fixed boundary conditions (Job et al., 2005). It is clear, however, that to incorporate ordered granular media into practical applications such as granular interfaces, it is important to model, study and understand how these highly discrete and strongly nonlinear media interact with flexible boundaries, e.g., when they are supported by spatially continuous structures or media. A natural step in this direction is to study the nonlinear interactions of ordered granular media with "non-standard" interfaces (boundaries), e.g., strings, membranes, shells or plates. There is a limiting number of previous works that considered the effects of linear flexible boundaries with ordered granular media.

Yang et al. (2011) theoretically, numerically and experimentally studied a 1D homogeneous granular chain in contact with a cylindrical elastic medium and found that the formation and propagation of reflected solitary waves from the interface is strongly influenced by the elastic modulus and geometry of the adjacent linear medium. In another work by Yang et al. (2012) a combined discrete element and finite element model was constructed to computationally investigate the interaction between a 1D homogeneous granular chain with a thin plate; in the same work a series of experiments was performed to confirm the computational predictions. The experimental results along with the numerical simulations confirmed the conclusion that the amplitude and speed of propagation of reflected solitary waves depend on the plate thickness and the size of the granules. In a recent work by Potekin et al. (2016), an iterative numerical algorithm was developed and applied to investigate the nonlinear wave scattering at the interface of 1D dimer granular chains with a linear string on an elastic foundation. Energy conservation within the system provided validation of this new numerical approach and thus ensured the accuracy of the numerical predictions. Following this, the effects of resonances or antiresonances in the granular dimer chains on the transmission and reflection of energy at the interface with the flexible boundary were studied and the physics of nonlinear wave scattering at the flexible boundary was investigated in detail.

Motivated by Potekin et al. (2016), in this work we study the nonlinear wave scattering at the interface of finite 1D granular dimer chains with a (linear) 2D rectangular membrane on elastic foundation. Apart from developing the computational algorithm that will enable the performance of this study, of emphasis will be the investigation on the effects of resonances and antiresonances in the dimers on the transmitted and reflected waves at the 2D flexible interface, as well as the study of possible localization phenomena (such as the formation of transient breathers (Potekin et al., 2016) in the neighborhood of the interface. The manuscript is structured as follows. In Section 2, we present the governing equations of the integrated granular dimer–membrane system and express them in non-dimensional form. In Section 3, we study the highly nonlinear and discontinuous wave scattering phenomena realized at the flexible boundary between dimer chain and the membrane. The effects of the dimer mass ratio and foundation stiffness on energy transmission, reflection and scattering at the chain-membrane interface are investigated also. In Section 4, we study the energy transmission between two granular chains through waves propagating in the flexible boundary. Finally, in Section 5 we summarize our observations and discuss possible future work.

2. Theoretical modeling

2.1. Mathematical model

In this work, we consider initially uncompressed 1D dimer (diatomic) granular chains composed of N linearly elastic, spherical, alternating "heavy" and "light" granules with Hertzian interaction, in contact to a membrane on a linear elastic foundation. The system depicted in Fig. 1 is considered in the initial part of this work where a computational algorithm is developed for studying the strongly nonlinear wave scattering at the flexible interface of the granular dimer with the membrane. At a later part of this work multiple granular chains in contact with the membrane are considered. The Hertzian contact assumptions (Nesterenko, 2001) are made to facilitate the corresponding analysis. In addition, gravity is neglected, and no plasticity or any dissipative effects are taken into consideration in our analysis. Also, in the absence of precompression granule separations and ensuing collisions between them can occur, rendering the acoustics highly discontinuous and strongly nonlinear.

When the spherical, linearly elastic granules are in Hertzian contact, the nonlinear Hertzian interaction force can be mathematically modeled by, $F = k\Delta_+^{3/2}$, where k is the coefficient of Hertzian interaction dependent on geometrical and material parameters of the interacting granules, $\Delta \geq 0$ is the distance approach between the centers of the interacting granules, and F the resulting con-

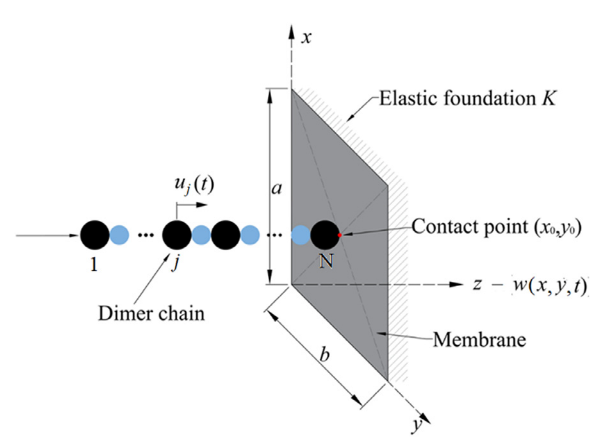

Fig. 1. Schematic of a dimer granular chain in contact with a membrane on elastic foundation.