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Robust uncertainty quantification in structural dynamics under scarce experimental modal data: A Bayesian-interval approach

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ABSTRACT

The accurate prediction of the dynamic behaviour of a complex component or system is often difficult due to uncertainty or scatter on the physical parameters in the underlying numerical models. Over the past years, several non-deterministic techniques have been developed to account for these model inaccuracies, supporting an objective assessment of the effect of these uncertainties on the dynamic behaviour. Still, also these methods require a realistic quantification of the scatter in the uncertain model properties in order to have any predictive value. In practice, this information is typically inferred from experiments. This uncertainty quantification is especially challenging in case only fragmentative or scarce experimental data are available, as is often the case when using modal data sets. This work therefore studies the application of these limited data sets for this purpose, and focuses more specifically on the quantification of interval uncertainty based on limited information on experimentally obtained eigenfrequencies. The interval approach, which is deemed to be the most robust against data insufficiency, typically starts from bounding the data using the extreme values in the limited data set. This intuitive approach, while of course representing the experiments, in general yields highly unconservative interval estimates, as the extreme realisations are typically not present in the limited data set. This work introduces a completely new approach for quantifying the bounds on the dynamic properties under scarce modal data. It is based on considering a complete set of parametrized probability density functions to determine likelihood functions, which can then be used in a Bayesian framework. To illustrate the practical applicability of the proposed techniques, the methodology is applied to the well-known DLR AIRMOD test case where in a first step, the bounds on the experimental eigenfrequencies are estimated. Then, based on a calibrated finite element model of the structure, bounds on the frequency response functions are estimated. It is illustrated that the method allows for a largely objective estimation of conservative interval bounds under scarce data.

1. Introduction

1.1. Context

The most elementary dataset in the characterisation of the dynamic behaviour of a complex system is the list of resonance frequencies, usually ranked from low to high, and covering the frequency range of interest which depends on the conditions

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of utilisation of the system. The other traditional modal parameters are the mode shapes, which are much more difficult to represent and sometimes also the damping values. The importance of each individual eigenfrequency in the overall dynamic response of the structure is expressed by the modal participation factor, which depends amongst other factors on the nature of the corresponding mode shape on and the position(s) of the point(s) of excitation. Not only in the structural response do the eigenfrequencies play a crucial role, also in numerical simulations the list of eigenfrequencies is the primary dataset for subsequent dynamic analysis of any kind. And when a finite element model needs to be validated, a dynamic analysis provides much more information than a static analysis, with the set of eigenfrequencies again the key target in the model updating procedure.

Both in experimental identification and in numerical simulation, traditional procedures for dynamic analysis are deterministic, which implies that every quantity, both on the input and on the output side, are represented by a unique number. Awareness is growing that non-determinism is an inevitable aspect of physical reality, for many reasons. Conditions of utilisation depend on time, temperature, humidity, degradation of the structure, …Each individual operator of the system may have a different user profile. Regardless of quality levels, different realisations of a nominally identical system never behave in exactly the same way. And also in experimental testing, several repetitions need to be done to identify the system characteristics as realistically as possible.

The common procedure for the experimental identification of resonance frequencies is a modal test. Even if standard data acquisition procedures call for a number of averages to be conducted, data analysis for each individual test leads to one set of resonance frequencies. Repetition of the test, even on the same realisation or definitely on another sample of a batch of nominally identical systems, or on a system which has been disassembled and reassembled, invariably produces a different set of resonance frequencies. The most complete and thus the most reliable information on the system requires a number of repetitions and a number of eigenfrequency datasets, which are represented by a number of histogrammes on individual eigenfrequencies. Such a histogramme is by definition a finite sample of a population of physically consistent representations of a non-deterministic system. Straightforward statistical analysis of each individual histogramme does not necessarily produce the most reliable representation of the system dynamics. This paper proposes a Bayesian approach for the experimental identification of the system dynamics, as it is represented by the limited dataset of eigenfrequencies.

1.2. Interval representation of eigenfrequency datasets

In structural dynamics applications, the estimation of the uncertainty in resonance frequencies of a structure is of large interest to designers who want to avoid disastrous resonance problems to occur during its service life. Such estimation based on experimental data is often very challenging given the cost associated with the necessary experimental campaigns. A possible solution is to apply interval methods for the quantification of this uncertainty. Indeed, interval calculation is finding its way in the world of structural dynamics and numerical modelling. More specifically, the interval finite element method has been well established [1–4] and implemented to determine intervals on output quantities of Finite Element (FE) models subject to interval uncertainties. Also, recent advances in surrogate modelling [5,6] (anti-)optimization [7–9] and interval field theory [10–14] further extends the applicability of interval analysis in numerical dynamical analysis. A next step towards practical application of this method is the capturing of uncertain input data in a suitable format by defining and possibly estimating intervals from available data [15–17]. Interval estimation concerns the capturing of the variability of a quantity x by an upper and a lower bound that effectively limit the possible values that x can take. Besides this, in between these bounds, all values are assumed to have a finite nonzero probability of occurring. However, no value of x has a defined probability, except those which fall out of the interval, as they have a probability equal to 0. Obviously, this means that before using interval quantification, it should be clear that acquiring probabilistic output (e.g. failure chances), is not an objective of the uncertainty analysis. This is for example the case in a worst-case scenario analysis, where the objective is to identify the lowest performing possible outcome (e.g. resonance frequencies) of a non-deterministic system and to ensure it is still sufficiently performing according to the demands. All quantities in the field of interest are theoretically bounded. Many quantities cannot drop below zero (eigenfrequencies, thicknesses, stiffnesses, etc.). However, their physical bounds are of greater interest, which are usually much narrower and therefore of greater practical use. Assume an uncertain quantity x and a total population of N. A total of n samples ($n \ll N$) of this population are available, in which a minimal value and maximal value $\left\langle \widehat{x_m} \mid \widehat{x_M} \right\rangle$ is observed. The objective is to find plausible values on the minimal and maximal value $\underline{x}$, and $\overline{x}$ within the entire population, and use this as bounds for the interval $x^I = \left\langle \underline{x} \mid \overline{x} \right\rangle$. Obviously, a first estimate could be to just take the observed bounds as estimate for the interval: $x^I = \left\langle \widehat{x_m} \mid \widehat{x_M} \right\rangle$, but for very small n, this can possibly be a very non-conservative result, with many samples in the total population falling out of the estimated interval. This renders the interval not useable for further reliability analysis, as too little variability of the quantity x is captured by the interval. The result should be conservative enough while still taking into account as much of the available information contained in the limited dataset as possible.

Section 2 discusses general aspects about Bayesian analysis and its application in possibilistic context. Section 3 reformulates Bayes's theorem in terms of interval bounds and Extreme Value Distributions and a parametrization of the generalize the PDF shape. Section 4 then introduces two ways to evaluate the likelihood function in interval Bayesian analysis. The last section discusses the application of this method to frequency data obtained from the DLR AIRMOD structure.