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Fast hypervolume approximation scheme based on a segmentation strategy☆

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ABSTRACT

Hypervolume indicator based evolutionary algorithms have been reported to be very promising in many-objective optimization, but the high computational complexity of hypervolume calculation in high dimensions restrains its further applications and developments. In this paper, we develop a fast hypervolume approximation method with both improved speed and accuracy than the previous approximation methods via a new segmentation strategy. The proposed approach consists of two crucial process: segmentation and approximation. The segmentation process recursively finds areas easy to be measured and quantified from the original geometric figure as many as possible, and then divides the measurement of the rest areas into several subproblems. In the approximation process, an improved Monte Carlo simulation is developed to estimate these subproblems. Those two processes are mutually complementary to simultaneously improve the accuracy and the speed of hypervolume approximation. To validate its effectiveness, experimental studies on four widely-used instances are conducted and the simulation results show that the proposed method is ten times faster than other comparison algorithms with a same measurement error. Furthermore, we integrate an incremental version of this method into the framework of SMS-EMOA, and the performance of the integrated algorithm is also very competitive among the experimental algorithms.

1. Introduction

Multiobjective optimization problems (MOPs) often involve multiple conflicting objectives to be optimized simultaneously in real-world applications. Due to the conflicting nature of the objectives, MOPs do not exist one single solution that is able to optimize all the objectives. Instead, they aim at finding a series of best possible trade-off solutions termed Pareto optimal solutions, whose objectives cannot be improved further more. The Pareto optimal solutions are known as the Pareto front (PF) in objective space and the Pareto set (PS) in decision space, respectively. Evolutionary multiobjective optimization (EMO) algorithms, as a class of population based search heuristics, have been successfully applied in various bi-objective and tri-objective optimization scenarios. Based on various acceptance rules of selecting offspring solutions, EMO algorithms

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could be generally divided to three branches: 1) Pareto dominance-based approaches, e.g. see [16], 2) Decomposition-based methods, e.g. see [12,22,23,30,31,46], and 3) Indicator-based algorithms, e.g. [5,48]. Pareto dominance-based algorithms rely on the relationship of Pareto dominance among solutions to approach the PF, Decomposition-based methods decompose the original problem into several subproblems and utilize scalarizing function to conduct selections in each subproblem. However, they encounter various kinds of difficulties when the number of objectives increases to more than three [11], which are known as many-objective optimization problems (MaOPs) [13,41]. In contrast to the two other groups of EMO algorithms, indicator-based algorithms use performance indicators to guide the evolution (*e.g.*, hypervolume, I_{SDE}) [5,29]. Prominent examples of this kind of EMO algorithms are IBEA [48], SMS-EMOA [5,32], MO-CMA-ES [26], and R2-EMOA [42], etc. In this paper, we mainly focus on the hypervolume indicator based EMO algorithms.

Hypervolume indicator is a volumetric measurement in geometry that calculates the union volume of a dominated region which was produced by a set of points. It was originally proposed as a set quality indicator called 'size of the space covered' to quantitatively compare the populations of different EMO algorithms [5,49]. This concept was later denoted as 'hypervolume measurement' [5]. Because hypervolume indicator has a significant attribute that keeps fully sensitive and effective to both Pareto dominance and population diversity [10,20], it is now one of the most important indicators in many-objective optimization.

Hypervolume indicator based EMO algorithms such as HypE [2], SMS-EMOA [5], FV-MOEA [27], HAGA [38] and MOP-SOhv [21], etc have been reported to be very promising in solving MaOPs. For instance, S Metric Selection Evolutionary Multi-Objective Algorithm (SMS-EMOA) [5,34], is designed to maximize the exact hypervolume of populations by means of discarding the least contribution solution. This idea in essence utilizes the hypervolume indicator to measure the quality of solutions. The superiority of SMS-EMOA has been verified by a plethora of research studies [33,47]. Unfortunately, it is still unpractical to apply SMS-EMOA to high-dimensional MaOPs due to the costly computational resources of hypervolume indicator calculation [4,7,8]. In [7], Bringmann and Friedrich proved that hypervolume calculation is a NP–complete problem. It is expected that no polynomial calculation algorithm exists since this would imply $NP = P$ [4,8].

Therefore, the main problem of this kind of EMO algorithms can be attributed to find a method that calculates the hypervolume indicator (or hypervolume measurement) efficiently. Without a fast and quality hypervolume calculation algorithm, the hypervolume based EMO algorithms would either have high computational complexity or compromised performance [2,9].

A lot of methods have been proposed to calculate the hypervolume indicator, and generally, two classes can be seen. The first class is the exact calculation method, which is mainly realized by means of the recursive computation. It divides the hypervolume calculation problem into several subproblems with the same property as the original problem and solves them respectively. This kind of method is featured by high accuracy and exponentially increasing complexity. Some influential algorithms of this kind include hypervolume by slicing objectives (HSO) [6,18] algorithm, HOY [3,24,36], Quick hypervolume (QHV) [39,40], HBDA [28] and Walking Fish Group (WFG) [14,37,43,44], etc. The second class is to estimate the hypervolume value by some statistical methods [1,7]. Monte Carlo simulation is a representative method, and its effectiveness in high dimensions has been proved by some performance analyses [35]. The first attempt in this direction is presented in [1]. Bringmann and Friedrich [7] later presented an efficient FPRAS (fully polynomial-time randomized approximation scheme) algorithm for hypervolume calculation. Its complexity is $O(dn/\epsilon)$ with the error of $\pm\sqrt{\epsilon}$. Though this method can reduce the complexity of hypervolume computation, the accuracy and running time have to be compromised. The main reason is the huge number of sampling points needed to reach the condition of dependable accuracy, which seriously influences the running time. Therefore, how to get it a further performance boost is still one of the major research problems.

In this paper, we propose a new hypervolume approximation method, which consists of two parts: segmentation and approximation. The basic idea of this method is to reduce the number of Monte Carlo sampling points by a new segmentation strategy. This segmentation strategy recursively segments the original geometric problem into a hypercube and several subproblems which are similar to the original problem. After segmentation, a modified Monte Carlo simulation is used to approximate these rest subproblems, which is called 'in corners' part. Due to the quite small size of these 'in corners' part, when applying modified Monte Carlo simulation, the number of sampling points would be much less than the original method. With the reduction of the number of sampling points, the running time is also reduced. This method is a combination of the exact calculation method and the approximation method, and thus, we call it 'partial precision and partial approximation'.

To better understand the proposed hypervolume approximation method, theoretical analysis about this method is given in this paper. Meanwhile, a series of experiments are conducted to systematically investigate its efficiency. To be specific, we compare the proposed method with three exact calculation methods (QHV, WFG, and HBDA) and an approximation method (FPRAS) on four widely-used test instances. Simulation results show that the proposed algorithm has low running time in high dimensional hypervolume calculation. The proposed algorithm can normally achieve about ten times faster than the FPRAS algorithm which is the most efficient algorithm in more than 10 dimensional space so far. Furthermore, we propose a method to find the solution that contributes the least to hypervolume in a population, which is often called the incremental version. We apply the proposed method into the framework of SMS-EMOA [5] and compare it with NSGA–III [15], MOEA/D [46], HypE [2], IBEA [48] and the incremental version of IWFG and exQHV in the framework of SMS-EMOA. By comparing, the proposed algorithm can achieve an acceptable performance in respect of both runtime and the quality of solutions.