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Accuracy of approximation operators during covering evolutions

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ABSTRACT

We express our concern over two issues in this paper: one is which characters are foundational for multitudinous covering-based rough models as generalizations of Pawlak's rough sets, i.e., which properties of covering-based rough sets guarantee the consistency of them and Pawlak's over partitions; the other is how to utilize this equivalence in simplifying diverse covering-based approximation operators. We demonstrate that covering-based rough models are equivalent to Pawlak's on partitions if they satisfy granule selection principles, which are weaker than a combination of contraction, extension, monotonicity, addition and granularity. In order to take advantage of their equivalence, we illustrate a method named "covering evolution" to change granules from given coverings to corresponding partitions for covering-based approximation operators. The evolutions can be divided into three steps: at the beginning, coverings are transformed into 1-neighborhood systems based on some quintessential neighborhood operators, then in the middle, more-refined 1-neighborhood systems are built by using "cores" from general topology, and at last, core systems (in fact, partitions) are extracted from these more-refined 1-neighborhood systems. We lay a strong emphasis on the variations in accuracy of six representative covering-based approximation operators during three evolutions, two of which preserve the accuracy of approximation operators. The investigation carried on covering evolutions helps us to establish the corresponding mapping relationship from covering spaces to partition spaces directly, and therefore provides a convenient method for making choice of covering-based approximation operators as they are consistent with the classical Pawlak's rough sets over partitions.

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1. Introduction

The pioneer work concerning rough set theory can be traced to Pawlak in 1982 [18]. Two definable subsets, namely, lower and upper approximations are assigned to each subset of a universe. Based on these crucial sets, more knowledge hidden in information systems can be expressed as decision rules [1,5,20,29]. As an effective tool to describe and extract useful information, rough set theory has been successfully applied in data mining, rule extraction, granular computing, process control, conflict analysis, medical diagnosis and other fields [13,15,17,19,28,38,41].

Pawlak's rough sets are extended to covering-based rough sets due to the limitation on equivalence relations [2,3,6–8, 16,21,23,25]. It is known that covering-based approximation operators are firstly presented by Zakowski [39]. Since then,

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increasing covering-based rough sets have appeared [40,42]. The dual approximation operators of Zakowski were analyzed by Pomykala. Twenty pairs of covering-based approximation operators were investigated and they were reduced to sixteen pairs in [22,32]. Moreover, the equivalence between covering-based rough sets and relation-based rough sets was also discussed in [24,34]. Tsang et al. demonstrated approximations and reducts with covering generalized rough sets [26]. A new rough set model was introduced by combining covering-based rough sets, fuzzy rough sets, and multi-granulation rough sets [37].

Neighborhood systems were introduced by Lin from the point of interior and closure in topology [9]. Actually, reflexive neighborhood systems are special coverings on universes. Some rough set models can be interpreted as ones established on the basis of different neighborhoods. Lin explored the applications of neighborhood systems and related approximations in knowledge bases as well as relational database [11,12]. Yao studied approximation retrieval models based on neighborhood systems [33]. Wu and Zhang derived six kinds of k-step neighborhood systems in [30]. Recently, D'eer et al. verified equalities of twenty four neighborhood operators and reduced them to thirteen different ones [4].

As we have seen, growing covering-based approximation operators have been defined and investigated. Observe that different covering-based approximation operators have different properties, a natural question is : as a generalization of Pawlak's rough sets, which properties are basic attributes for covering-based approximation operators? i.e., which properties guarantee the equivalence between covering-based approximation operators and Pawlak's over partitions? Additionally, it should be noted that very little work has been carried on simplifying them in some way. In consideration of the identity of Pawlak's rough sets and some representative covering-based rough models on partitions, it is worth transforming coverings into partitions for the later. In this work, we make a first attempt to establish the corresponding relationship between coverings and partitions on universes by means of covering evolutions. Of course, we hope that some "nice" properties should be preserved during the evolutions. The remainder of this paper proceeds as follows: Section 2 describes a slice of related definitions. Section 3 discusses conditions under which covering-based rough models are equivalent to Pawlak's on partitions. Section 4 demonstrates the covering evolutions and checks the accuracy of six pairs of approximation operators step by step. In Subsection 4.1, we introduce three neighborhood operators with transitivity or symmetry to turn common coverings into 1-neighborhood systems. In Subsection 4.2, by means of cores adopted from topology, we refine 1-neighborhood systems constructed in Subsection 4.1. In Subsection 4.3, we study the properties of the cores of refiner 1-neighborhood systems generated in Subsection 4.2. The key point of our research in Section 4 is to reveal the variation of approximation operator accuracy over the covering evolutions. Results of this paper and our future work are summarized in Section 5.

2. Definitions

In this paper, the universe X is assumed to be a finite set. Unless otherwise stated, the symbol $\mathcal{U}$ means a covering of X. We denote a pair of approximation operators by (L, H), where L refers to the lower approximation operator and H is the upper approximation operator.

Minimal description: For each $x \in X$, we define a collection named minimal description of x as follows [40]:

$Md_{\mathcal{U}}(x) = \{U : x \in U, U \in \mathcal{U} \wedge (\forall V \in \mathcal{U} \wedge x \in V \wedge V \subseteq U \Rightarrow U = V)\}$.

Minimal inclusion : For each $x \in X$, we call the following subset of X minimal inclusion of x [40]:

$Mi_{\mathcal{U}}(x) = \bigcap\{U : x \in U, U \in \mathcal{U}\}$.

For each element x in X, one associates it with a subset $n(x)$ of X named a *neighborhood* of x. A *neighborhood system* $NS(x)$ is a nonempty family of neighborhoods of x. A neighborhood system of X is the collection of $NS(x)$ for all $x \in X$. A neighborhood system is a *1-neighborhood system* (*1-NS* in short) if each element of X has exactly one neighborhood [31].

1-NSs described below have been considered in [31]:

reflexive 1-NS: $x \in n(x)$ for all $x \in X$;

symmetric 1-NS: $x \in n(y) \Rightarrow y \in n(x)$ for all $x, y \in X$;

transitive 1-NS: $[y \in n(x), z \in n(y)] \Rightarrow z \in n(x)$ for all $x, y, z \in X$.

Given that a 1-NS $\{n(x) : x \in X\}$ on X, for each $x \in X$, the *core* of $n(x)$ refers to $cn(x) = \{y \in X : n(y) = n(x)\}$. The family $\{cn(x) : x \in X\}$ is defined as the *core system* of $\{n(x) : x \in X\}$.

A *neighborhood operator* is a mapping $\mathcal{N}$: $X \to \mathcal{P}(X)$, where $\mathcal{P}(X)$ is the collection of subsets of X [32]. A neighborhood operator $\mathcal{N}$ on X is reflexive (symmetric, transitive respectively) if 1-NS $\mathcal{N}(X) = \{n(x) : x \in X\}$ is reflexive (symmetric, transitive respectively).

The following is a list of some properties for a reflexive 1-NS $\{n(x) : x \in X\}$:

Property A. $x \in cn(x)$ for each $x \in X$.

Property B. $cn(x) \subseteq n(x)$ for each $x \in X$.

Property C. $cn(x) = n(x)$ for each $x \in X$ if $\{n(x) : x \in X\}$ is a partition.

Property D. $cn(x) \subseteq n(y)$ for $x, y \in X$ whenever $cn(x) \cap n(y) \neq \emptyset$ and $\{n(x) : x \in X\}$ is symmetric or transitive (Lemma 3.10 in [35]).

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