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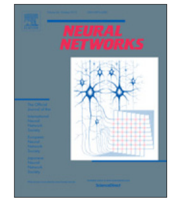

Effects of infinite occurrence of hybrid impulses with quasi-synchronization of parameter mismatched neural networks

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ABSTRACT

This article is deeply concerned with the effects of hybrid impulses on quasi-synchronization of neural networks with mixed time-varying delays and parameter mismatches. Hybrid impulses allow synchronizing as well as desynchronizing impulses in one impulsive sequence, so their infinite time occurrence with the system may destroy the synchronization process. Therefore, the effective hybrid impulsive controller has been designed to deal with the difficulties in achieving the quasi-synchronization under the effects of hybrid impulses, which occur all the time, but their density of occurrence gradually decrease. In addition, the new concepts of average impulsive interval and average impulsive gain have been applied to cope with the simultaneous existence of synchronizing and desynchronizing impulses. Based on the Lyapunov method together with the extended comparison principle and the formula of variation of parameters for mixed time-varying delayed impulsive system, the delay-dependent sufficient criteria of quasi-synchronization have been derived for two separate cases, viz., $T_a < \infty$ and $T_a = \infty$. Finally, the efficiency of the theoretical results has been illustrated by providing two numerical examples.

1. Introduction

Over the past three to four decades, the dynamical behaviors of neural networks, such as stability, periodic attractors, bifurcation, and chaotic attractors have been extensively investigated due to their potential applications in pattern recognition, optimization, signal processing, and associative memories. Moreover, chaos synchronization of chaotic neural networks has been a fascinating problem since the pioneering work of Pecora and Carroll (1990). Till date, various types of synchronization of neural networks have been studied, such as projective synchronization (Chen & Cao, 2012), quasi-synchronization (Tang, Park, & Feng, 2018), lag-synchronization (Hu, Yu, Jiang and Teng, 2010), lag quasi-synchronization (Huang, Li, Huang, & Han, 2013) etc., due to their theoretical importance and practical applications in secure communication (Lakshmanan et al., 2018), tracking control (Yang, Feng, Feng and Cao, 2017), and image encryption (Wen, Zeng, Huang, Meng, & Yao, 2015). In most of the synchronization schemes, master and slave systems are identical. But in the real world, it is common to have parameter mismatches between the systems and their existence may slow the speed of convergence or even destroy the synchronization. Therefore, many researchers have paid their attention towards finding the implications of parameter mismatches to the synchronization process, see the references He et al. (2015), Huang et al. (2013) and Tang et al. (2018).

One more thing is worth mentioning that time-delay in neural network is inevitable due to the finite-time speed of signal propagation, and finite time required for information processing, etc. Presence of time-delay in the system may result in instability or stability of the system's trajectory depending on the value of delay. Therefore, many significant results have been found regarding the implications of time-delay on systems' dynamics (Balasubramaniam & Vembarasan, 2012; Chen & Cao, 2012; Rahman, Blyuss, & Kyrychko, 2015; Yang, Huang and Li, 2017; Zhou & Cai, 2018; Zhu & Cao, 2010). Mainly, there are two types of time-delay, viz., discrete and distributed time-delays, which play a significant role to change the dynamical behaviors of the systems. Basically, the structure of a neural network is a spatial nature because of having so many parallel pathways with a variety of axon sizes and lengths, so it is logically meaningful to consider distributed delay in its modeling.

In order to achieve synchronization, most of the systems' networks need some external forces, namely, controller. Only

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a few networks exist in nature and the artificial world; those could achieve synchronization by adjusting their systems' parameters (Arenas, Diaz-Guilera, Kurths, Moreno, & Zhou, 2008; Matheny et al., 2014). Many effective control functions have been developed for investigating synchronization and stability problems of neural networks, such as adaptive control (Cao, Wang, & Sun, 2007), intermittent control (Zhang, Huang, & Wei, 2011), quantized intermittent pinning control (Xu et al., 2018), integral sliding mode control (Shi, Zhu, & Qin, 2014), and impulsive control (Chen, Shi, & Lim, 2018; He, Qian, Cao, & Han, 2011; He et al., 2015; Li, Ho, & Lu, 2017; Li, Lou, Wang, & Alsaadi, 2018; Lu, Ho, & Cao, 2010; Tang et al., 2018; Yang, Huang, & Zhu, 2011; Yi, Feng, Wang, Xu, & Zhao, 2017), etc. In the viewpoint of engineering applications, the intermittent and impulsive controllers are very efficient in reducing the cost of control and the amount of transmitted information due to their discontinuity in nature. The working mechanism of both controllers could be described in two points: (i) intermittent control is activated interval-wise, i.e., in some intervals it works but in others it does not work, for example in Hu, Yu et al. (2010), Cheng et al. had applied periodically intermittent control to achieve the lag-synchronization of neural networks with mixed time-varying delays, where controller works only for periodically partitioned intervals. (ii) impulsive controller is activated only at discrete points which can be found in He et al. (2011), where the controller works only at the impulsive moments of the impulsive sequence $\{t_k\}_{k=1}^{\infty}$ to control the states of slave system so that they get synchronized with the states of master system within a synchronization error bound. It is clear from the working mechanism of both controllers that impulsive controller is more efficient than an intermittent controller to reduce control cost and amount of transmitted information.

In recent years, very progressive efforts have been reported in Chen and Cao (2012), Chen et al. (2018), He, Qian, and Cao (2017), He et al. (2011), He et al. (2015), Lu et al. (2010), Tang et al. (2018), Yang et al. (2011), Yi et al. (2017), Zhang, Li, and Huang (2017) and Zhang, Tang, Fang, and Wu (2012) to make impulsive controller more effective in investigating stability and synchronization of less conservative non-linear dynamical systems. Generally, impulses are characterized in to three categories: synchronizing impulses ($|\mu_k| < 1$), desynchronizing impulses ($|\mu_k| > 1$), and inactive impulses ($|\mu_k| = 1$). Most of the works using impulsive controller have been done for synchronizing impulses and desynchronizing impulses, separately. There are a few articles in which synchronization or stability problems have been investigated by using synchronizing and desynchronizing impulses, simultaneously. For example, in Zhang et al. (2012), the authors have studied the stability of delayed neural networks with the effects of stabilizing ($|\mu_k| < 1$) and destabilizing impulses ($|\mu_k| > 1$), simultaneously. It is shown in the article that the simultaneous effects of stabilizing and destabilizing impulses could not affect the stability of delayed neural networks, adversely, if stabilizing impulses can prevail over the influence of destabilizing impulsive effects. Lu et al. (2010) have developed a new concept of average impulsive interval and studied a unified synchronization of impulsive dynamical systems with simultaneous effects of synchronizing and desynchronizing impulses. The main motivation behind the concept was the idea of average dwell time (Hespanha & Morse, 1999) and the intuition to enhance the results like (Guan, 2018; Hu, Jiang and Teng, 2010; Li & Rakkiyappan, 2013; Li & Song, 2013; Pu, Liu, Jiang, & Hu, 2015; Sheng & Zeng, 2018), which are based on supremum and infimum of impulsive intervals. Later on, many results on the synchronization of impulsive dynamical systems using average impulsive interval are published, see Chen et al. (2018), He et al. (2011), He et al. (2015), Li, Shi, and Liang (2019), Lu et al. (2010), Tang et al. (2018), Yang et al. (2011), Yang, Lu, Ho, and Song (2018) and Yi et al. (2017). Unfortunately, in Chen et al. (2018), He et al. (2011), He et al. (2015), Li et al. (2019), Lu et al. (2010), Tang et al. (2018), Yang et al. (2011) and Yi et al. (2017), authors could not discuss the effects of hybrid impulses over synchronization scheme when impulses occur infinitely, but sparsely, i.e., $T_a = \infty$. In the real world, there exists an impulsive sequence in which impulses occur all the time, but the length of their impulsive intervals increases with time. Such type of impulses does not essentially adversely affect the synchronization of the coupled neural networks. Inspired from this fact, authors in Wang, Li, Lu, and Alsaadi (2018) proposed two new concepts of average impulsive interval $T_a = \lim_{t\to\infty} \frac{t-s}{N_\zeta(t,s)}$ and average impulsive gain $\mu = \lim_{t\to\infty} \frac{|\mu_1|+|\mu_2|+\cdots+|\mu_{N_\zeta(t,s)}|}{N_\zeta(t,s)}$, and derived the unified synchronization criteria for an array of coupled neural networks with hybrid impulses under the influence of $T_a = \infty$.

Motivated from all the discussions mentioned above, in this article, the quasi-synchronization of different neural networks with mixed time-varying delays has been investigated for $T_a = \infty$ using the hybrid impulsive controller. The new concepts proposed in Wang et al. (2018) are adopted in this article to deal with the situation of $T_a = \infty$ and the simultaneous effects of synchronizing and desynchronizing impulses. The main contributions of this article can be described in the following points.

(1) Different from Tang et al. (2018), we have designed a hybrid impulsive controller for time-varying impulses. Further, for dealing with the simultaneous effects of different types of impulses ($|\mu_k| < 1$ and $|\mu_k| > 1$), the concept of average impulsive gain has been adopted from Wang et al. (2018).

(2) We have successfully applied the modified version of the average impulsive interval to study the case of infinite but sparse occurrence of impulses, i.e., the situation when impulsive interval $T_a = \infty$, on quasi-synchronization of neural networks with parameter mismatches and mixed time-varying delays.

(3) Using some mathematical techniques and the extended comparison principle for time-varying delayed impulsive differential equation combined with the formula for the variation of parameters, the delay-dependent criteria for quasi synchronization of the neural networks with mixed time-varying delays and parameter mismatches have been derived for $T_a < \infty$ and $T_a = \infty$. Also, the small domains of convergence containing the origin have been obtained into which the trajectories of the controlled neural networks are converging globally exponentially at the respective rate of convergence.

The remaining portion of this article is organized as follows. In Section 2, the problem formulation for the models of neural networks is done and then some important preliminaries, such as definitions, assumptions, and lemmas which are needed to prove the main results of this article, are listed. For two different cases, one for $T_a < \infty$ and another for $T_a = \infty$, the sufficient criteria of quasi-synchronization between different neural networks with mixed time-varying delays are derived in Section 3. Two examples are considered in Section 4 to validate the theoretical results proposed in this article. Finally, the overall conclusions of this work have been drawn in Section 5.

Notations: The following standard notations will be used in this article. $\mathbb{R}$ is a set of real numbers. $\mathbb{R}^n$ denotes the Euclidean space of column vectors of dimension n. $\lambda_{max}(.)$ presents the largest eigenvalue of a square matrix. $\mathbb{R}^{n\times n}$ denotes the Euclidean space of square matrices of order n. The notation "T" means transpose of a matrix or a vector. $\|.\|$ is 2-norm which is defined as $\|y\| = \sqrt{\sum_{1=1}^{n} y_i^2}$ for a column vector y and $\|A\| = \sqrt{\lambda_{max}(A^TA)}$ for a square matrix A. $[t]$ indicates the least integer of number less than or equal to t.