Computers & Graphics xxx (xxxx) xxx

Contents lists available at ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag

Technical Section

Descriptions and evaluations of methods for determining surface curvature in volumetric data

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A R T I C L E I N F O

Article history:
Received 17 June 2019
Revised 11 November 2019
Accepted 14 November 2019
Available online xxx

Keywords:
Curvature
Implicit surfaces
Volume data
Direct volume rendering

A B S T R A C T

Three methods developed for determining surface curvature in volumetric data are described, including one convolution-based method, one fitting-based method, and one method that uses normal estimates to directly determine curvature. Additionally, a study of the accuracy and computational performance of these methods and prior methods is presented. The study considers synthetic data, noise-added synthetic data, and real data. Sample volume renderings using curvature-based transfer functions, where curvatures were determined with the methods, are also exhibited.

1. Introduction

Surface curvature is a common shape descriptor that is used in a wide range of applications including isosurface extraction [1] and visualization [2], molecular field feature extraction and analysis [3], location of features in seismic data [4,5], object recognition [6] and segmentation [7], analysis of anatomical abnormalities [8], direct volume rendering [9,10], contour enhancement [10], etc. Curvature, simply put, describes the amount by which a surface "bends" away from its tangent plane [11], with the magnitude indicating the amount of bending and the sign indicating the direction of the bending relative to the tangent plane's normal. The magnitude and sign may differ depending on the direction they are measured within the tangent plane. However, the directions in which the maximum and minimum curvatures occur are always orthogonal. We denote the maximum and minimum curvature values as κ_1 and κ_2, respectively. κ_1 and κ_2 are the *principal curvatures*, and the directions in which they occur are the *principal directions* ([12], Chapter 2). The principal curvatures are two commonly used curvature values. Additionally, two other curvature quantities used in some tasks, including direct volume rendering [10] and segmentation [7], can be computed from these two: the Gaussian curvature, $H = \kappa_1 \times \kappa_2$, and the mean curvature, $K = (\kappa_1 + \kappa_2)/2$.

Here, our focus is on the determination of the principal curvatures at each point within a volume, where the curvature values at a point describe the bending, at that point, of the implicit surface corresponding to the isosurface defined by the value of the point. Each volume thus potentially contains a large number of surfaces, depending on the number of values exhibited within the volume. Mathematically, such principal curvatures can be found as the first two eigenvalues of $\nabla\vec{n}$, where $\vec{n}$ is the surface normal at the point (i.e., the normalized gradient of the volume). Since this definition of curvature makes use of first and second derivatives of a mathematical function that describes a surface, using it to calculate curvature in discrete data, such as in volumetric datasets, requires strategies to directly estimate these derivatives or closely-related quantities. (N.B., some methods, described later, estimate closely-related quantities rather than the derivatives themselves). Moreover, for curvature determination in real, sensed data the necessary estimations are further complicated by the presence of noise.

In this paper, we describe three methods (two primary methods and a third method that is a variant of one of the two) developed to determine curvature in regular rectilinear grid volumetric datasets (here forward: "volume data"). These methods are adapted from methods for determining curvature in range images. In addition, we examine the accuracy and computational performance of these methods and of seven classic, existing methods [7,9,10,13–16].

Our examinations provide extensive comparison of curvature determination method accuracies when operating on noise-free and noise-added synthetic volume data and on real, sensed volume data. Prior work of Wernersson et al. [15] also compared some such curvature determination methods in terms of accuracy. However, unlike the Wernersson et al. work, which only compared methods based on their curvature formulations and excluded other differences in processing, our studies here consider entire

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https://doi.org/10.1016/j.cag.2019.11.003
0097-8493/© 2019 Published by Elsevier Ltd.

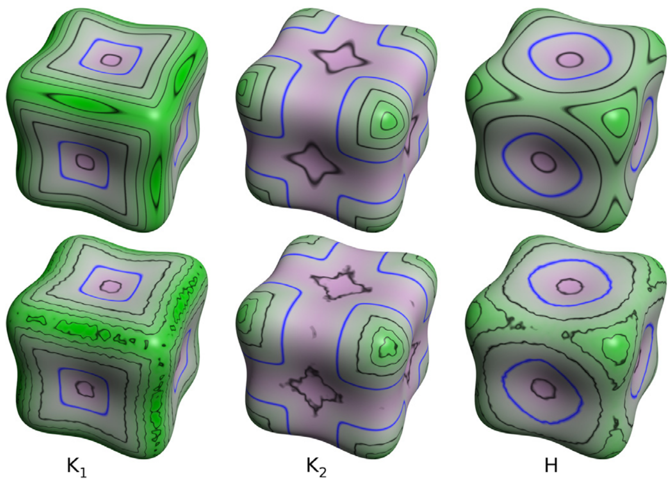

Fig. 1. Renderings of $f(x, y, z) = x^2 + y^2 + z^2 - x^4 - y^4 - z^4$ using curvature-based color transfer functions based on a variety of curvature quantities. Columns from left-to-right: κ_1, κ_2, and H. On the top row, exactly computed curvatures were used. On the bottom row, curvatures were perturbed by either +10% or -10% of the curvature value at each point, demonstrating that even a modest level of deviation from correct curvatures may result in notable changes in renderings.

methodologies, including specific filters and smoothing operations used in each method. Additionally, we include here evaluation of several additional determination methods and analysis of the computational performance of all the methods. This work is an extension of our previous conference paper [17]. Here, two additional curvature determination methods and additional datasets are considered. Additional analysis has also been performed and integrated here, including an evaluation of the impact of variations in parameter selection as well as plots and evaluations of global average error values. Our work here also provides, for comparison, a number of direct volume renderings (DVRs) produced using the curvature outputs of each determination method. Such renderings have previously been noted for their ability to enhance the utility of volume rendering [10], however, as shown in Fig. 1, even a modest amount of deviation from the correct curvature values can markedly impact rendering quality. (A rendering similar to the top part of Fig. 1 was previously presented in [10].) Our comparison renderings thus provide a visual indication of how selection of the curvature determination method can impact the common real-world task of volume visualization. To our knowledge, our work here is the most extensive study on the impact of the curvature determination method on rendering quality when using curvature-based direct transfer functions.

This paper is organized as follows. Section 2 (Section 2) provides background information on surface curvature. Existing methods to determine curvature in volume data are also discussed. Section 3 presents the three volume extensions of existing methods for determining curvature in range images. Section 4 describes performance comparisons of these methods and the prior methods. Section 5 presents visual comparisons of DVRs produced using curvature-based color transfer functions where curvatures were determined by said methods. Section 6 concludes the paper.

2. Previous work

Here, we describe curvature calculation generally as well as methods previously developed for determining curvature in volume datasets. First, we introduce our notation. (u, v, w) denotes a grid (or sample) point within the volume, where $0 \leq u < N_u$, $0 \leq v < N_v$, $0 \leq w < N_w$ for a volume of size $N_u \times N_v \times N_w$. The value at point (u, v, w) is denoted $f(u, v, w)$; f represents the underlying function that generates the volume. Consequently, f_u represents the partial derivative of f in the u direction. We additionally denote the gradient as $\vec{g} = [f_u\ f_v\ f_w]^T$, the normal as $\vec{n} = \frac{\vec{g}}{\|\vec{g}\|}$, a Hessian as $\mathbf{H}$, and identity matrices as $\mathbf{I}$. We denote both point locations and matrices using upper case, bold symbols.

2.1. Curvature calculation

As mentioned in the introduction, curvature in volumetric data can be computed mathematically as the first two eigenvalues of $\nabla\vec{n}$. However, such a strategy is problematic in practice. First, even when f is known, this computation requires finding the partial derivatives of a normalized gradient, which can be a tedious process for humans and even impossible for mathematics software packages (e.g., [18] found that some expressions are beyond the capabilities of computer algebra systems, and estimated derivatives must be used instead). Second, with sampled data, where f is unknown, determining change in estimated, normalized values is required, which hinders use of direct convolution-based derivative estimates [10]. As a result, multiple curvature formulations have been reported in the literature, with some for curvature determination from a known f ([19]), where derivatives need not be estimated but are computed directly from f, some focused on easing determination of complicated curvature-related functions that arise from a known f ([18]), and still others useful in determining curvature from an unknown f ([7,10]), where derivatives are first estimated and curvatures are then determined from these estimated derivatives. The focus of our work here is determination of curvature from an unknown f. Consequently, many of the prior strategies for curvature determination, described next, involve both derivative estimation schemes and their own formulation for curvature.

2.2. Prior strategies

Next, we describe prior methods for determining curvature in volume data. Many methods use a strategy of estimating derivatives (via convolution along each axis or fitting) and determining curvature using those estimated derivatives. A variant strategy is to avoid derivative estimation by exploiting various geometric properties of curvature (e.g., the method of Hladøuvka et al. [9] avoids explicitly estimating first derivatives and instead computes normals from tangents estimated by triplets of points). Some methods discussed here require a choice of one or more parameters, and in general the best parameter choices vary depending on the type of data considered (e.g., noise-free, real, containing fine features, etc.). In our discussions of the methods here, we also note our chosen parameters used in our tests presented later. For methods where existing recommended parameter choices exist, our reports here are based on those (except where noted otherwise). For others, we have attempted to choose parameters that provide reasonable accuracy across a variety of types of data (based on testing each method with a variety of parameter values and data types).

2.2.1. Taylor Expansion convolution (TE)

The curvature determination method of Kindlmann et al. [10] is based on separable convolutions that estimate first and second partial derivatives from which curvatures are ultimately determined. The convolutions use filters developed according to the framework of Möller et al. [20], which allows for a given accuracy and continuity.

The Kindlmann et al. method first applies these filters at every data point in the volume to estimate the derivatives. Then, the estimates of f's first and second derivatives are used to determine normals, gradients, and Hessians. From these, three quantities are

Please cite this article as: J.D. Hauenstein and T.S. Newman, Descriptions and evaluations of methods for determining surface curvature in volumetric data, Computers & Graphics, https://doi.org/10.1016/j.cag.2019.11.003