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An efficient iterative method for solving multiple scattering in locally inhomogeneous media

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Highlights

- An efficient iterative method is proposed for solving multiple scattering problem in locally inhomogeneous media.
- At each iteration, only single scattering problems need to be solved. An effective way is introduced to handle the communication between scatterers.
- The convergence of the iterative method is proved by using the compactness of involved integral operators.

Abstract

In this paper, an efficient iterative method is proposed for solving multiple scattering problem in locally inhomogeneous media. The key idea is to enclose the inhomogeneity of the media by well separated artificial boundaries and then apply purely outgoing wave decomposition for the scattering field outside the enclosed region. As a result, the original multiple scattering problem can be decomposed into a finite number of single scattering problems, where each of them communicates with the other scattering problems only through its surrounding artificial boundary. Accordingly, they can be solved in a parallel manner at each iteration. This framework enjoys a great flexibility in using different combinations of iterative algorithms and single scattering problem solvers. The spectral element method seamlessly integrated with the non-reflecting boundary condition and the GMRES iteration is advocated and implemented in this work. The convergence of the proposed method is proved by using the compactness of involved integral operators. Ample numerical examples are presented to show its high accuracy and efficiency.

Keywords: Multiple scattering; Inhomogeneous media; Iterative method; Spectral element method; Non-reflecting boundary condition; GMRES iteration

1. Introduction

The acquaintance of many physical phenomena and engineering processes can be significantly enhanced by accurately simulating the multiple scattering problems involving configurations of many obstacles. Typically, for

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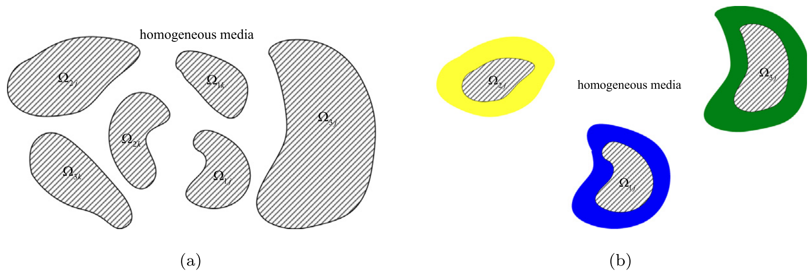

Fig. 1.1. Configurations of multiple scattering. (a): Scatterers embedded in homogeneous media; (b): Well-separated scatterers with inhomogeneous media in colored area. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

two-dimensional time-harmonic acoustic multiple scattering in inhomogeneous media, we consider the Helmholtz equation of the form

$$\Delta u(\boldsymbol{x}) + \kappa^2 n^2(\boldsymbol{x}) u(\boldsymbol{x}) = 0, \quad \text{in} \ \ \mathbb{R}^2 \setminus \Omega, \tag{1.1}$$

where $u = u^{\mathrm{sc}} + u^{\mathrm{in}}$ is the total field, κ is the wave number, $n^2(\boldsymbol{x})$ is the index of refraction, Ω is a region occupied by M impenetrable scatterers in $\mathbb{R}^2$, see Fig. 1.1. The scattering field u^{sc} satisfies the Sommerfeld radiation condition

$$\frac{\partial u^{\mathrm{sc}}}{\partial r} - \mathrm{i}\kappa u^{\mathrm{sc}} = o\big(r^{-1/2}\big), \quad \text{as} \ \ r := |\boldsymbol{x}| \to \infty. \tag{1.2}$$

On the boundaries of the scatterers, the Dirichlet, Neumann or Robin boundary conditions can be imposed according to different materials of the scatterers. Here, let M_1, M_2, M_3 be the number of scatterers with Dirichlet, Neumann and Robin boundary conditions, respectively, and denote by Ω_{1j}, Ω_{2j}, Ω_{3j} the jth scatterer in each group. Accordingly, we denote the domain of all obstacles and its boundary by

$$\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3, \quad \partial\Omega = \partial\Omega_1 \cup \partial\Omega_2 \cup \partial\Omega_3 \quad \text{with} \quad \Omega_i = \bigcup_{j=1}^{M_i} \Omega_{ij}, \tag{1.3}$$

for $i = 1, 2, 3$ corresponding to the Dirichlet, Neumann and Robin boundary conditions, respectively. For notational convenience, we express these three types of boundary conditions on the scatterers as

$$\mathscr{B}_i u = 0, \quad \boldsymbol{x} \in \partial\Omega_i, \quad i = 1, 2, 3, \tag{1.4}$$

where

$$\mathscr{B}_1 = \mathcal{I}, \quad \mathscr{B}_2 = \frac{\partial}{\partial \boldsymbol{n}}, \quad \mathscr{B}_3 = \frac{\partial}{\partial \boldsymbol{n}} + h\mathcal{I}. \tag{1.5}$$

Here, $\mathcal{I}$ is the identity operator, $\boldsymbol{n}$ is the unit outward normal on $\partial\Omega_2$ and h is a given function defined on $\partial\Omega_3$.

It is known that analytic solutions for wave scattering problems from multiple arbitrary shaped obstacles embedded in inhomogeneous media are not available. Partially for this reason, many early works are mostly restricted to cylindrical and spherical obstacles embedded in homogeneous media, where the modal expansions of the scattered fields play an essential role (cf. [1–5]). We highlight that the reader-friendly monograph by Martin [6] was largely concerned with time-harmonic waves with multiple obstacles and with exact methods including separation of variables, integral equations and T-matrices, but only the last chapter is concerned with some numerics.

Among limited works for multiple scattering problems with general bounded scatterers (compared with intensive studies of single scattering problems), the boundary integral method is one of the methods of choice. By reformulating a scattering problem into an integral equation on the boundary of scatterers (cf. [7–9]), numerical methods (e.g., boundary element methods) have been developed based on the Galerkin or collocation formulations (cf. [10,11]). Very recently, the boundary integral method with fast multipole acceleration and hybrid numerical-asymptotic boundary element method have been investigated for relatively low frequency (cf. [12]) and high frequency problems (cf. [13,14]), respectively. However, it is noteworthy that the boundary integral method relies