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A geometrically exact isogeometric beam for large displacements and contacts

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Abstract

This work discusses an efficient formulation of a geometrically exact three-dimensional beam which can be used in dynamical simulations involving large displacements, collisions and non-linear materials. To this end, we base our model on the shear-flexible Cosserat rod theory and we implement it in the context of Isogeometric Analysis (IGA). According to the IGA approach, the centerline of the beam is parameterized using splines; in our work the rotation of the section is parameterized by a spline interpolation of quaternions, and time integration of rotations is performed using the exponential map of quaternions. Aiming at an efficient and robust simulation of contacts, we propose the adoption of a non-smooth dynamics formulation based on differential-variational inequalities. The model has been implemented in an open-source physics simulation library that can simulate actuators, finite elements, rigid bodies, constraints, collisions and frictional contacts. This beam model has been tested on various benchmarks in order to assess its validity in non-linear static and dynamic analysis; in all cases the model behaved consistently with theoretical results and experimental data.

Keywords: IGA; Beam; Contact; Multibody; Non-smooth dynamics

1. Introduction

Deformable three-dimensional beams withstanding large displacements can be found in many scenarios of practical interest, this is the case of the blades in a helicopter rotor, for instance, or the case of the torsion bar in a car suspension, or the case of a flexible robot arm. In the last decades, similar problems motivated extensive research in the area of fast, robust and reliable computer methods for the simulation of beams, and most of those methods are based on Finite Element (FE) discretizations.

Recently, the Isogeometric Analysis (IGA) computational approach gained increased popularity as it blends the concept of spline interpolation in the context of FE. While traditional FE methods discretize the continuum using finite elements that share end nodes, the IGA approach uses a single spline with several nodes. Indeed, nowadays most CAD tools are based on splines of NURBS or B-spline type, hence one of the relevant advantages of IGA is that these geometries can be imported directly in the simulation, whereas FE methods require an intermediate

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pre-processing step in order to generate a mesh approximation of the original geometry [1]. Moreover, conventional finite elements using Lagrange polynomials feature C^0 continuity at nodal points regardless of their order, while spline functions of order p provide C^{p-1} continuity at all non-multiple knots. This means that for the same amount of degrees of freedom, IGA shows better robustness and accuracy compared to C^0-continuous FE [2].

Although the most obvious application of IGA is in the modeling of structural elements which can map to a line parameterization, such as cables and beams, the same concept can be extended to generic PDEs involving membranes, shells and volumes [3,4]. Among the vast literature on IGA, we cite in passing the developments on hierarchical refinement [5], the generalization to T-splines [6], the application to fluid dynamics [7] and to contact problems [8,9].

In this paper we use IGA to implement a geometrically exact three-dimensional beam based on the Cosserat rod theory, hence capable of arbitrary large rotations and displacements [10]. Non-linear geometric effects in beams have been studied extensively in computational mechanics and different approaches have been put forward: for example a straightforward method still used nowadays is based on reusing linear finite elements developed for infinitesimal-strain conventional beams of Euler–Bernoulli or Timoshenko type, by updating a local corotational reference frame that can have large displacements and rotations [11,12]. In contrast to this, but at the cost of a more sophisticated formulation, the *geometrically exact beam model*, also referred to as Simo–Reissner beam model, draws on the theory of 1D Cosserat continua [13] and leads to a general formulation that makes no assumption on the amount of rotations and that allows finite strains, including shear and torsion [14–16]. The Cosserat rod theory does not require that cross-sections must be orthogonal to the tangent of the centerline, hence it can account for shear effects similarly to the Timoshenko beam theory, of whom it can be considered a generalization to the 3D case [17]. In fact, Reissner, Kirchhoff–Love, Timoshenko and Euler–Bernoulli beams can be interpreted as special cases of Cosserat rods.

Many references on IGA-based beams can be found in literature: for instance IGA for Kirchhoff–Love and Euler–Bernoulli beams are discussed in [18] and [19], straight and curved 2D Timoshenko beams are discussed in [20], to name a few. Similarly to FE beams, numerical locking artifacts can affect IGA beams when shear is taken into account. In this context, a collocation approach was used in [21,22] to obtain a locking-free spatial IGA beam.

Contact between three-dimensional rods has been dealt in literature by various authors. Among the first results, in [23] a frictionless point-wise contact formulation was developed between pairs of cables with circular section, later extended to the frictional case [24]. The problem of contact between beams of rectangular section, that might generate multiple contact points between the edges, has been discussed in [25,26]. The issue of self-contact between beams has been studied in [27,28], a topic of great importance if one needs to simulate problems such as the tightening of knots, knitting machines or winches. Most contact models in literature are based on point-based formulations where the position of contact points is obtained by solving local sub-problems of distance minimization between curved geometries [29]. However this approach is near singular when beams intersect with small contact angles: in [30,31] an efficient method has been proposed that can solve this issue. In [32] the simulation of frictional contact and self collision is discussed for a class of efficient IGA-based Cosserat rods.

Similarly to [33], we address the case of a three-dimensional Cosserat rod of high generality, hence accounting for shear, torsion, geometric and material non-linearity. As an alternative to penalty-based beam contact formulations as presented in [34], we express it with a formalism that fits well in a time stepper for non-smooth multibody dynamics, using the theory of Differential Variational Inequality (DVI). To our knowledge, this is the first time an IGA formulation has been used in the framework of DVI non-smooth dynamics; the benefit being the fact that DVI formulations provide a robust and stable way to simulate contact problems even with large time-steps and many simultaneous contacts.

Early work on non-smooth dynamics can be traced back to the seminal research of Jean–Jacques Moreau on measure-differential inclusions, a special type of DVI [35,36]. The non-smooth nature of dynamical problems originates from various phenomena, most often it is a consequence of the Coulomb friction model and impulsive collisions at contact points; instead than regularizing the contact forces – a conventional method that leads to smooth but stiff differential problems – Moreau proposed to embed such set-valued force laws directly in the formulation. This can be done at the cost of assuming that velocities can be discontinuous, hence accelerations are considered as distributions of vector signed measures. Since then, different authors contributed to the field of non-smooth dynamics, see for example [37–39]. The robustness and stability of DVI methods motivated their use in scenarios