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Seamless integration of design and Kirchhoff–Love shell analysis using analysis-suitable unstructured T-splines

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Abstract

Analysis-suitable T-splines (ASTS) including both extraordinary points and T-junctions are used to solve Kirchhoff–Love shell problems. Extraordinary points are required to represent surfaces with arbitrary topological genus. T-junctions enable local refinement of regions where increased resolution is needed. The benefits of using ASTS to define shell geometries are at least two-fold: (1) The manual and time-consuming task of building a new mesh from scratch using the CAD geometry as an input is avoided and (2) C^1 or higher inter-element continuity enables the discretization of shell formulations in primal form defined by fourth-order partial differential equations. A complete and state-of-the-art description of the development of ASTS, including extraordinary points and T-junctions, is presented. In particular, we improve the construction of C^1-continuous non-negative spline basis functions near extraordinary points to obtain optimal convergence rates with respect to the square root of the number of degrees of freedom when solving linear elliptic problems. The applicability of the proposed technology to shell analysis is exemplified by performing geometrically nonlinear Kirchhoff–Love shell simulations of a pinched hemisphere, an oil sump of a car, a pipe junction, and a B-pillar of a car with 15 holes. Building ASTS for these examples involves using T-junctions and extraordinary points with valences 3, 5, and 6, which often suffice for the design of free-form surfaces. Our analysis results are compared with data from the literature using either a seven-parameter shell formulation or Kirchhoff–Love shells. We have also imported both finite element meshes and ASTS meshes into the commercial software LS-DYNA, used Reissner–Mindlin shells, and compared the result with our Kirchhoff–Love shell results. Excellent agreement is found in all cases. The complexity of the shell geometries considered in this paper shows that ASTS are applicable to real-world industrial problems.

Keywords: Isogeometric analysis; Analysis-suitable T-splines; Extraordinary points; Optimal approximation; T-junctions; Geometrically nonlinear Kirchhoff–Love shells

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1. Introduction

In engineering applications, shell geometries are often created in computer-aided-design (CAD) programs using different types of spline functions, such as non-uniform rational B-splines (NURBS) [1] and T-splines [2]. In order to generate complex geometries, the tensor-product structure of NURBS requires gluing together many different NURBS surfaces called patches. This procedure has at least two shortcomings: (1) Superfluous control points are needed due to the tensor-product structure of each patch [3] and (2) watertight geometries are difficult to obtain in most cases [4]. T-splines, through the use of T-junctions and extraordinary points, overcome the aforementioned shortcomings of NURBS [3,4]. T-junctions remove the tensor-product constraint of NURBS patches, thus enabling the placement of control points only where needed. Extraordinary points enable the representation of geometries with arbitrary topology using only one T-spline surface. The unstructured nature of T-splines also provides an alternative to the use of trimmed NURBS surfaces [5,6].

When it comes to performing numerical simulations of shell structures in computer-aided-engineering (CAE) programs, the current spline representations used in CAD programs are not necessarily suitable for analysis. As a result, a new mesh needs to be built from scratch using the original CAD geometry as an input. This process is time consuming, taking up to 80% of the "design to analysis" cycle in certain engineering applications [7,8]. Moreover, significant geometry modifications are often inevitable, thus jeopardizing the reliability of the analysis results. C^0 Lagrange polynomials, the standard basis functions in the finite element method [9], are frequently used to create the computational mesh. This reduces the smoothness of the original spline surface.

Isogeometric analysis (IGA) was introduced with the goal of developing a seamless integration between CAD and CAE programs when it comes to geometry representation. As a first step, a NURBS patch was shown to be a suitable basis for analysis in [10–13]. In contrast with the C^0 inter-element continuity of Lagrange polynomials, the higher inter-element continuity of a NURBS patch brings the following advantages: higher-order partial differential equations can be solved in primal form [14–18], enhanced robustness in solid mechanics is obtained [19], enhanced accuracy in spectrum analysis is achieved [20], H^1-conforming discretizations that are either divergence-conforming or curl-conforming can be derived in a straightforward manner [21–24], and partial differential equations can be collocated in strong form [25,26]. As opposed to a NURBS patch, which is directly analysis-suitable, T-junctions are not necessarily analysis-suitable [27]. To remedy this, a subset of T-splines called analysis-suitable T-splines (ASTS), including T-junctions but not extraordinary points, was defined which maintains all the important geometric and mathematical properties of a NURBS patch [28–35]. When at least C^1 inter-element continuity is imposed, the use of multi-patch NURBS or T-splines with extraordinary points does not necessarily lead to spaces with optimal approximation properties in analysis [36–38]. A construction of extraordinary points with at least C^1 inter-element continuity that results in optimal convergence rates with respect to the mesh size h for second- and fourth-order linear elliptic problems was recently developed in [39], thus enabling the extension of the ASTS definition to include both T-junctions and extraordinary points.

NURBS-based IGA was applied to Kirchhoff–Love shells in [15,40–50], Reissner–Mindlin shells in [51–55], solid-shell elements in [56–60], a hierarchic family of linear shells [61], and a shell formulation that blends Kirchhoff–Love theory with Reissner–Mindlin theory [62]. ASTS-based IGA, including T-junctions but not extraordinary points, was recently applied to Kirchhoff–Love shells in [63]. Triangular Loop and quadrilateral Catmull–Clark subdivision surfaces are another appealing alternative to integrate geometric modeling and shell analysis [64–70]. In the neighborhood of extraordinary points, however, the resulting basis functions obtained with subdivision surfaces are non-polynomial, which (1) complicates the numerical integration [71,72] and (2) harms the approximation order of the spaces [72,73].

In this work, we describe a detailed blueprint for the construction of smooth non-negative bi-cubic ASTS on unstructured meshes. This description is self-contained, complete, and general enough to handle all cases of interest in practice where one needs to work with meshes containing both extraordinary points and T-junctions. Extraordinary points enable generation of quadrilateral meshes for geometries with arbitrary topologies, while T-junctions allow us to locally increase the spline space resolution to one required for the purpose of analysis. Augmenting the classical ASTS construction (on locally structured meshes with T-junctions) with a novel extraordinary point treatment, we are able to achieve optimal convergence rates with respect to not only the mesh size but also the square root of the number of degrees of freedom for second- and fourth-order linear elliptic problems. We believe our description of the unstructured T-spline technology with extraordinary points improves upon and supersedes previous presentations in the literature, such as [74], which was deficient in its inability to achieve optimal convergence rates in the presence