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Isogeometric boundary element analysis for two-dimensional thermoelasticity with variable temperature

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ABSTRACT

This work is devoted to numerical analysis for two-dimensional thermoelasticity problems with temperature change by using the isogeometric boundary element method (IGABEM). The present IGABEM, which is highly attractive, possesses advantages of the isogeometric analysis with NURBS and boundary element method. We derive the theoretical formulations in terms of the IGABEM and apply it to thermal stress analysis. We examine the performance and accuracy of the proposed approach through numerical test cases which include steady-state uniform and non-uniform temperature change. The computed results are compared with the reference solutions which were derived from analytical or finite element methods. We also investigate the convergence of the present approach in modeling thermal stress problem.

1. Introduction

Many engineering structures operate under high temperature conditions such as gas turbines, diesel engines, and nuclear power plants. The existence of temperature field could significantly alter material properties of the structures or components and generate thermal stress as the temperature changes. The thermal stress plays a critical role that could lead to the damage of such structures. Investigation of the temperature field and thermal stress affecting the structures under heating has become an important topic in structural analysis.

The theory of thermal stress has been well developed in the literature. Although some closed–form analytical solutions are available, advanced numerical methodologies are more effective in solving engineering problems with general geometries and/or boundary conditions. In the past several decades, many numerical methods have been introduced to deal with thermal stress problems, such as finite element method (FEM) [1,2], boundary element method (BEM) [3–5], radial integration method (RIM) [6], singular boundary method (SBM) [7], generalized finite difference method (GFDM) [8], and extended finite element method (XFEM) [9].

The FEM has been widely used in various scientific and industrial communities. However, the existing gap between computer–aided design (CAD) and finite element analysis (FEA) is well-known as a critical issue, and generating computational model in general is time-consuming. In addition, element-based polynomial approximation used in the FEA induces the discretization errors, especially for complex structure. The recently developed isogeometric analysis (IGA) has become a powerful numerical approach, see e.g., [10,11] and references therein, which can eliminate such drawback. One of the underlying characteristics of the IGA lies in the use of the CAD basis functions as shape functions, and the CAD control points as the mesh nodes in FEA, thus the IGA unifies the fields of the CAD and FEA. Besides the advantages of the traditional FEM, the merits of the IGA lie in the fact that the IGA owns several desirable features, for instance, exact geometry, high accuracy of the solutions, high order continuity, and without traditional meshing procedure, see [10,11]. The IGA has been successfully implemented in many areas of engineering and science, see e.g., [12–21].

In CAD, non-uniform rational B–splines (NURBS) describe the boundaries of structure only, so one of the crucial steps in the IGA is to generate solid analysis models based on boundaries, and at present it is still a difficult task in generating such solid analysis models, especially for complex structures. In contrast, only the boundaries of the domain are meshed in the BEM, it has resulted in a new combined approach between the IGA and BEM (called as IGABEM), and the key issue of creating solid analysis models required in the IGA is no-longer required. Simpson et al. [22] first introduced the IGABEM to solve the two-dimensional (2D) elasticity, and proposed the method for dealing with singular integrals. Later, Simpson et al. [23] further explained the

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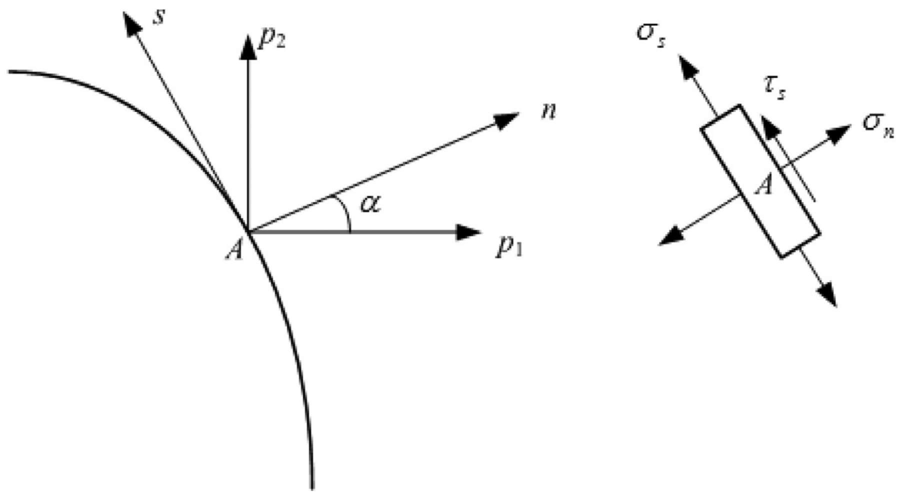

Fig. 1. Schematic of stress components at boundary points.

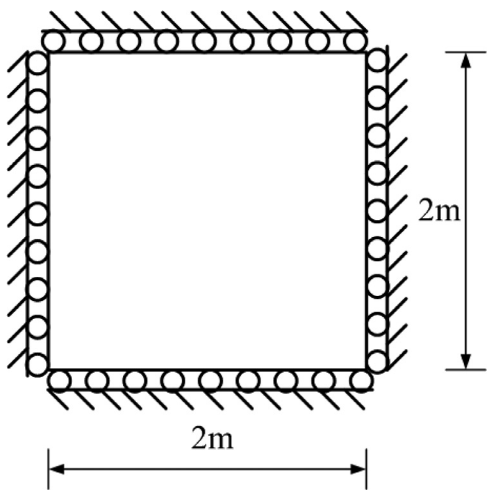

Fig. 2. Geometry and boundary conditions of a square plate.

operation of the program, and presented the open source MATLAB program code. Gu et al. [24] developed local bivariate B-spline functions and successfully applied them to the IGABEM for three-dimensional (3D) potential problem, avoiding the calculation of a large number of singular integrals and near singular integrals. Gong et al. [25,26] used power series expansion method to improve the accuracy of singular integrals, while near singularities in 2D and 3D potential problems can be removed or weaken with the aid of exponential transformation method. Han et al. [27] approximated the singular integral terms by using Taylor series polynomials expressions, and derived the semi-analytical expressions through a serious of integration by parts. Beer et al. [28] proposed a new IGABEM method using trimmed NURBS, which is simple and effective. Wang et al. [29] presented multi-patch nonsingular IGABEM with trimmed elements. Because the coefficient matrix of the IGABEM is full as that in BEM, it occupies a large amount of memory in computer, and the computational efficiency is low, and fast algorithm for the IGABEM emerges a key research issue. Takahashi and Masumoto [30] used fast multipole algorithm to solve 2D Laplace problem. Simpson and Liu [31] proposed a black-box fast multipole method based on T-spline.

The NURBS control points have been used as design variables for structural shape optimization problems. Therefore, the design model, optimization model, and analysis model can be uniformly described with the NURBS. In that sense, the optimized boundary in general is smooth. Hence, the IGABEM is highly suitable for structure shape optimization. Li and Qian [32] adopted the IGABEM to optimize the shape of 2D and 3D elasticity problems. Lian et al. [33] used regularized IGABEM to optimize the shape of two-dimensional elasticity problems, avoiding the calculation of strong singular integrals, jump terms and shape derivatives. Sun et al. [34] used particle swarm optimization combined with the IGABEM to optimize the structure shape, eliminating the complicated sensitivity analysis process.

Beer et al. [35,36] studied 2D and 3D elastic-plastic inclusions using the IGABEM. Peng et al. [37] simulated crack growth with NURBS-based IGABEM. Sun et al. [38] modeled crack propagation by using IGABEM based on Bézier extraction. Peake et al. [39] solved the 2D Helmholtz problem using the extended IGABEM. Coox et al. [40] solved Helmholtz problem with isogeometric indirect BEM. May et al. [41] solved 2D stationary magnetic and magneto-mechanical field problems using a hybrid method of IGA finite element and IGABEM. In our previous work, we used the IGABEM to investigate 2D steady heat transfer problems [42]. The numerical results show the advantages of the present IGABEM as it offers acceptable solutions and owns several desirable features of a powerful and efficient numerical method.

Some major desirable features of the IGABEM can be summarized as follows: (a) it has the exact representation of geometries; (b) a traditional meshing process is avoided; (c) the high accuracy can be obtained because of the use of the NURBS basis; (d) the advantages of the IGA and BEM are possessed simultaneously; and (e) the volume parameterization is not required, which is one of the key issues in the isogeometric finite element method. In this paper, we further extend the IGABEM to solve thermal stress problems with varying temperature. We derive the formulations of IGABEM for thermoelasticity analysis with variable temperature, and present the main numerical implementation. Numerical results confirm high accuracy of the developed IGABEM for thermoelasticity. In addition, the computer codes are provided and that should be helpful for other researchers

The rest of the manuscript is structured as follows. Section 2 briefly introduces the thermal stress problem. The formulation of IGABEM for thermal stress analysis is described in Section 3. Section 4 presents the main numerical implementation. In Section 5, several numerical examples are considered and the computed results are compared with the analytical solution or FEM solution. In Section 6, we discuss several major conclusions observed from the analysis.

2. Problem statement

In absence of body forces, the equilibrium differential equation in elasticity can be written as

$$\sigma_{ij,j} = 0 \tag{1}$$

where σ_{ij} is the stress tensor.