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2-D DOA estimation via correlation matrix reconstruction for nested L-shaped array

Yunlong Yang [a,b], Xingpeng Mao [a,b,*], Yuguan Hou [a,b], Guojun Jiang [a,b]

[a] School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin, China
[b] Key Laboratory of Marine Environmental Monitoring and Information Processing, Ministry of Industry and Information Technology, Harbin, China

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ABSTRACT

For a nested L-shaped array (N-LsA) composed of two orthogonal nested subarrays, the self-difference co-array of each nested subarray is hole-free, whereas cross-difference co-arrays between subarrays have holes. Due to the existence of holes, virtual cross-correlation matrices with increased degree of freedoms (DOFs) can not be constructed from cross-difference co-arrays, which will degrade the performance of direction of arrival (DOA) estimation. To overcome this problem, a high resolution two-dimensional (2-D) DOA estimation algorithm is exploited for N-LsA in this paper. Specifically, by using oblique projection operators, filled cross-difference co-arrays can be achieved by filling the holes, and virtual cross-correlation matrix will be obtained. Then the virtual correlation matrix of the N-LsA, which consists of virtual cross-correlation matrices and virtual autocorrelation matrices given by filled self-difference co-arrays, is reconstructed for 2-D DOA estimation. Additionally, the proposed algorithm contains an automatic angle-pairing procedure and can handle underdetermined DOA estimation. The estimation error, Cramér-Rao bound and computational complexity are derived. Simulation results show that the proposed algorithm offers substantial performance improvement over the existing algorithms.

1. Introduction

Direction of arrival (DOA) estimation is an important application of antenna arrays in radar, communication, sonar and many other fields [1–4]. It is well known that an uniform linear array (ULA) with N antennas can detect at most $N-1$ received signals by using subspace-based methods, e.g., MUSIC algorithm [5]. However, by exploiting the difference co-array provided from the original array, the number of signals which can be resolved will be increased dramatically and the estimation performance of algorithms will be improved significantly. To this end, nested arrays and coprime arrays with only N antennas have been investigated to provide self-difference co-array with $O(N^2)$ degree of freedoms (DOFs) for one-dimensional (1-D) DOA estimation [6, 7]. Particularly, for a two-level nested array with N antennas, its self-difference co-array is filled (i.e., no holes), and can provide $(N^2-2)/2+N$ DOFs for 1-D DOA estimation by using MUSIC algorithm [6].

In practical applications, many geometrical configurations have been exploited for 2-D DOA (i.e., elevation and azimuth angles) estimation, such as L-shaped array (LsA) [8], parallel linear array [9], circular arrays [3], rectangular arrays [10] and etc. Particularly, as the LsA composed of two orthogonal linear subarrays has good DOA estimation performance and is easy to implement, a number of approaches for 2-D DOA estimation have been investigated based on it, e.g., [8,11–13] and references therein. In [11], an approximate maximum likelihood method for 2-D DOA estimation is proposed with arbitrarily distributed arrays (including LsA). It can obtain a good performance within a few iterations when the initial 2-D DOA are proper. To reduce the computational burden, some methods perform 2-D DOA estimation via two independent 1-D DOA estimations [8,12]. Particularly, the LsA in [8] consists of one ULA and one sparse linear array, which can provide larger aperture compared with the LsA composed of two ULAs. Nevertheless, both the 2-D DOA estimation algorithm in [8,12] can only estimate $(N-1)$ sources with N physical antennas in each axis. A combined real-valued subspace based method for 2-D DOA estimation has been explored with an improved LsA [13], where the numbers of antennas of two ULAs respectively are N and M. The maximum resolvable source number of the proposed method in [13] is $\min(M-1, 2N-1)$ with $M>N$, which can increase resolvable source number compared with previous work [8,11,12].

* Corresponding author at: School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin, China.
E-mail addresses: yangyl@hit.edu.cn (Y. Yang), mxp@hit.edu.cn (X. Mao), yuguanhou@hit.edu.cn (Y. Hou), jiangguojun@hit.edu.cn (G. Jiang).

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In order to inherit the advantages of the nested array (or coprime array) and LsA, the nested LsAs (N-LsAs) (or coprime LsAs) composed of two two-level nested arrays (or coprime arrays) have been exploited [14–16]. It should be noted that for a LsA, the angle α between an incident signal and one linear subarray is usually different from the angle β between the incident signal and the other one. Thus the cross-difference co-array with (α, β), which is from cross-correlation matrix between these two subarrays, has holes. Hence the virtual cross-correlation matrix can not be constructed for DOA estimation. Therefore, the existing estimation algorithms, which only operate filled self-difference co-arrays of two nested (or coprime) subarrays separately [14,15], degrade their performances. Also, estimation performance of existing algorithm [16], which combines hole-free self-difference co-arrays and cross-difference co-arrays with holes, may be degraded. Besides, the effect of noise is not considered in the aforementioned algorithm [16].

It is noted that the holes in the self-difference co-arrays of coprime arrays can be filled by the techniques such as nuclear norm minimization [17–19] and positive definite Toeplitz completion [19–21] recently. The requirement of those techniques is that the resulting virtual correlation matrix has an Hermitian Toeplitz structure. However, due to $\alpha \neq \beta$, the above virtual cross-correlation matrix is not a Hermitian Toeplitz matrix. Thus such techniques will not be used for filling holes in the cross-difference co-array mentioned above.

In this paper, to overcome this problem based on N-LsA, we propose a 2-D DOA estimation algorithm for high resolution, which uses both filled self- and cross-difference co-arrays by filling the holes. Specifically, the virtual autocorrelation matrices, which are from the filled difference co-arrays of the two-level nested subarrays in the N-LsA, are used to perform 1-D DOA estimation separately. By using cross-correlation matrix, an automatic angle pairing is given to pair estimated angles. Then oblique projection (OP) operators are built with these paired angles to produce the component of each signal in the holes. Hence the holes will be filled and therefore the filled cross-difference co-array can be achieved. As a result, the virtual cross-correlation matrix can be reconstructed, which has the same dimension of the virtual autocorrelation matrix. The virtual correlation matrix of N-LsA, which combines virtual cross-correlation matrices and virtual autocorrelation matrices, will be reconstructed for 2-D DOA estimation.

The rest of this paper is organized as follows. Section 2 reviews the signal model, difference co-array and two-level nested array. In section 3, the proposed algorithm is provided for N-LsA. Section 4 provides the performance analysis of the proposed algorithm. In section 5, numerical examples are given to show the effectiveness of our proposed algorithm. Finally, Section 6 contains conclusions.

In this paper, scalars are denoted by lowercase italic letters, e.g., a. Vectors are denoted by italic boldface lowercase letters, e.g., $\boldsymbol{a}$. Matrices are denoted by italic boldface capital letters, e.g., $\boldsymbol{A}$. We list some notational conventions which will be used in the paper.

- $\lfloor a \rfloor$: a number rounded to the nearest integer a and $\lfloor a \rfloor \leq a$
- $|a|$: absolute value of a
- $\langle \boldsymbol{a} \rangle_i$: the item of a vector $\boldsymbol{a}$ corresponding to the signals at the ith location of an array
- $diag(\boldsymbol{a})$: a diagonal matrix whose diagonal elements are given by $\boldsymbol{a}$
- $\boldsymbol{A}^*$: complex conjugate of $\boldsymbol{A}$
- $\boldsymbol{A}^{\mathrm{T}}$: transpose of $\boldsymbol{A}$
- $\boldsymbol{A}^{\mathrm{H}}$: conjugate transpose of $\boldsymbol{A}$
- $\mathrm{E}\{\boldsymbol{A}\}$: mathematical expectation of $\boldsymbol{A}$
- $vec(\boldsymbol{A})$: vectorizing matrix $\boldsymbol{A}$
- $||\boldsymbol{A}||$: the Euclidean norm of $\boldsymbol{A}$
- $\boldsymbol{A} \odot \boldsymbol{B}$: Khatri-Rao product of $\boldsymbol{A}$ and $\boldsymbol{B}$
- $\boldsymbol{A} \otimes \boldsymbol{B}$: Kronecker product of $\boldsymbol{A}$ and $\boldsymbol{B}$
- $\boldsymbol{I}_N$: an $N \times N$ identity matrix
- $\boldsymbol{O}_{N\times M}$: an $N \times M$ zero matrix

2. Preliminaries

In this section, we will give a signal model, introduce the concept of difference co-array, review two-level nested array and N-LsA, and briefly introduce the oblique projection operator.

2.1. Signal model

We consider that K far-field narrowband signals from 1-D directions $\{\varphi_k, k = 1, 2, \ldots, K\}$ impinge on a linear array. The minimum distance between antennas is half-wavelength, i.e., $\lambda/2$. So the signal model $\boldsymbol{x}$ of a linear array at time t is given as

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{1}$$

where $\boldsymbol{s}(t) = [s_1(t)\ s_2(t) \ldots s_K(t)]^{\mathrm{T}}$ is the signal vector, and $\boldsymbol{A} = [\boldsymbol{a}(\varphi_1)\ \boldsymbol{a}(\varphi_2)\ \ldots\ \boldsymbol{a}(\varphi_K)]$ is the array manifold matrix with $\boldsymbol{a}(\varphi_i)$ denoting the spatial steering vector of the ith signal. Furthermore, the white Gaussian noise $\boldsymbol{n}(t)$ is assumed to be uncorrelated with the signals. Also, the signals are assumed to be temporally white and uncorrelated with each other.

Then, the autocorrelation matrix of $\boldsymbol{x}(t)$ is written as

$$\boldsymbol{R}_{\boldsymbol{x}} = \mathrm{E}\left\{\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)\right\} = \boldsymbol{A}\boldsymbol{R}_{\boldsymbol{s}}\boldsymbol{A}^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I}_N, \tag{2}$$

where signal autocorrelation matrix $\boldsymbol{R}_{\boldsymbol{s}} = diag\left(\sigma_1^2\ \sigma_2^2\ \ldots\ \sigma_K^2\right)$ is a diagonal matrix with σ_i^2 being the ith signal's power, and σ_n^2 is the noise power.

2.2. Difference co-array

For a given array with N antennas, let $\vec{\boldsymbol{S}}_i$ denote the position vector of the ith antenna. We have [6]

$$\boldsymbol{D} = \pm\{\vec{\boldsymbol{S}}_i - \vec{\boldsymbol{S}}_j\}, \quad \forall i, j = 1, 2, \cdots, N. \tag{3}$$

It is shown that set $\boldsymbol{D}$ contains some duplicate items. Removing repeated items (after their first occurrence), let $\boldsymbol{D}_u$ contain the distinct elements of the set $\boldsymbol{D}$.

Subsequently, the *self-difference co-array* of a linear array is defined as the array whose antenna positions are given by the set $\boldsymbol{D}_u$ from (3). The linear array configuration indicates that $\vec{\boldsymbol{S}}_i$ degenerates to a scalar quantity.

When $\vec{\boldsymbol{S}}_i$ belongs to one linear array X with N antennas and $\vec{\boldsymbol{S}}_j$ belongs to the other linear array Y with M antennas, (3) can be rewritten as [22]

$$\boldsymbol{D} = \pm\{\vec{\boldsymbol{S}}_{X,i} - \vec{\boldsymbol{S}}_{Y,j}\}, \quad 1 \leq i \leq N, 1 \leq j \leq M. \tag{4}$$

The *cross-difference co-array* related to these two linear arrays is defined as the array whose antenna positions are given by the set $\boldsymbol{D}_u$ from (4).

2.3. Two-level nested array and N-LsA

For a two-level nested array with $N = N_1 + N_2$ antennas [6], the dense sub-ULA has N_1 antennas with antenna distance $d_1 = \lambda/2$, and the sparse sub-ULA has N_2 antennas with antenna distance $d_2 = (N_1 + 1)d_1$, as shown in Fig. 1. If N is odd, $N_1 = (N-1)/2$ and $N_2 = (N+1)/2$. And if N is even, $N_1 = N_2 = N/2$. Hence, the antenna positions are at [6]

$$\boldsymbol{S}_T = \{0, \cdots, N_1 - 1, N_1, \cdots, [N_2(N_1 + 1)] - 1\}d_1. \tag{5}$$