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# Multi-vehicle Cooperative Control for Load Transportation ⋆

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**Abstract:** This work proposes a cooperative control solution to the problem of transporting a suspended load using multiple quadrotor vehicles. The problem is addressed for two quadrotors, with a methodology that can be generalized for any number of quadrotors. A dynamic model of the system is developed considering a point-mass load, rigid massless cables, and neglecting aerodynamic effects of the cables. The concept of differential flatness is explored and a new set of flat outputs, which can be used to fully characterize the state of the system, is proposed. A nonlinear Lyapunov-based controller in cascaded form is derived, by defining adequate mappings between the cable tension vectors and the quadrotor thrust vectors and exploring the analogy with the problem of controlling a single quadrotor. Simulation results are presented for tracking of load trajectories. Comparisons are made with a free-flying quadrotor control scheme to highlight the enhanced performance of the proposed scheme.



*Keywords:* nonlinear control, differential flatness, cascaded systems, autonomous vehicles, slung-load transportation

## 1. INTRODUCTION

In recent years, there has been a rise in the use of Unmanned Aerial Vehicles (UAVs) for various applications. In particular, quadrotor UAVs have received a lot of attention due to their high maneuverability in 3D environments, high thrust to weight ratio and reduced mechanical complexity. Applications include coverage of media events (photography and filming), infrastructure inspections, and mobile sensor networks, to name a few. Among these applications, load transportation is a topic that has been explored for several years. As highlighted in Villa et al. (2018), it has gained importance in both civilian and military applications, taking advantage of the vehicles ability to describe precise trajectories for transportation of fragile cargo.

Research on slung-load transportation goes from path-planning to control system design and estimation problems. Several results available in the literature use the concept of differential flatness, which defines a class of dynamical systems for which all states and inputs can be described as functions of the so-called flat output and its time derivatives. This property has been explored to address both motion planning and tracking control problems. For example, the work in Sreenath et al. (2013) shows that a system comprised of a single quadrotor and a load connected by a inelastic massless cable is differentially flat and extends also the definition for the full hybrid system that results from considering the case when the cable is not taut. This concept is further developed in Sreenath and Kumar (2013), considering a rigid body load and proving differential flatness for 3 or more quadrotors. The work in Kotaru et al. (2017) develops further from Sreenath and Kumar (2013) by describing the cable via a mass-spring model to account for its elasticity. Although the resulting system is not differentially flat, a geometric controller is proposed and convergence is proven for the reduced dynamics via single perturbation theory. In Cabecinhas et al. (2019), a nonlinear Lyapunov-based trajectory tracking controller is proposed for the case of a single quadrotor and suspended load, which relies on expressing the system in an adequate form for application of the backstepping technique, providing asymptotic stabilization with a large region of attraction. The work in Lee et al. (2013) applies geometric control to address the problem of slung-load transportation using an arbitrary number of quadrotors and a point-mass load, proposing an inner-outer loop control structure.In Lee (2018) this control method is extended to consider a rigid-body load. Pereira et al. (2016) analyse the full model of a single quadrotor and a load and define the domains for the inputs and angular velocity of the cable where the cables remain taut and the quadrotor's thrust points outwards to avoid compression forces on the cables. In Pereira and Dimarogonas (2017) the problem of

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controlling two drones with a slung-load is addressed by separating the problem into three decoupled systems and controlling each one separately: one for position, another for the yaw angles and a final one for the plane defined by the two cables.

In this work, we propose a cooperative control scheme for multi-vehicle load transportation, exploring the definition of a new set of flat outputs. These include the position of the load and a set of angles that completely define the cable directions and consequently the relative positions of the quadrotors. Explicit mappings between the cable tension vectors and the quadrotor thrust vectors are then explored to define an error system, starting with the load position error, progressing to the cable direction errors, and ending with quadrotors' thrust direction error. The resulting closed-loop system takes the form of a cascaded system, whose origin is shown to be asymptotically stable.

The paper is organized as follows. In Section 2, the model of the system to be adopted and the problem to be solved are introduced. In Section 3, the system is analyzed for differential flatness and the control scheme is derived. Simulation results are presented in Section 4 and Section 5 summarizes the contents of the paper.

## 2. PROBLEM STATEMENT

Consider two quadrotors, with masses $m_{Q_1}$ and $m_{Q_2}$, connected by massless, rigid links with lengths $l_1$ and $l_2$ to a point-mass load with mass $m_l$. An inertial reference frame $\{I\}$ is introduced, along with two body reference frames $\{B_i\}$ where $i \in \{1, 2\}$, each one fixed to the center of mass of quadrotor $i$. The inertial reference frame has its z axis pointing downwards, in the direction of the gravity vector, which is assumed to be constant. As shown in Fig. 1, let the direction of each cable $i$ be described by a unit vector $\boldsymbol{q}_i \in \mathbb{S}^2$, where $\mathbb{S}^2 = \{\boldsymbol{q}_i \in \mathbb{R}^3 \mid \|\boldsymbol{q}_i)\| = 1\}$, expressed in the inertial reference frame and centered at the origin of the body frame $\{B_i\}$. The load position expressed in $\{I\}$ is defined as $\boldsymbol{x}_l$ and the orientation of quadrotor $i$, or more specifically, the rotation matrix from $\{B_i\}$ to $\{I\}$ is defined as $R_{Q_i} \in \mathbb{SO}(3)$, where $\mathbb{SO}(3) = \{RR^T = I \mid det(R) = I\}$ denotes the Special Orthogonal Group of order 3.

Within this setting, the control objective consists of designing a control law to achieve tracking of a desired trajectory for the load, i.e. guarantee that the load position $\boldsymbol{x}_l(t)$ converges asymtptotically to $\boldsymbol{x}_{l_d}(t)$.

Given the rigid link assumption, it immediately follows that

$$\boldsymbol{x}_{Q_i} = \boldsymbol{x}_l - l_i \boldsymbol{q}_i \quad , \tag{1}$$

where $\boldsymbol{x}_{Q_i}$ is the position of quadrotor $i$ expressed in $\{I\}$ (see Fig. 1). By Newton's Law, the following expression can be obtained for the total accelerations of the load and quadrotors

$$\begin{cases} m_l \ddot{\boldsymbol{x}}_l = -T_1 \boldsymbol{q}_1 - T_2 \boldsymbol{q}_2 + m_l g \boldsymbol{e}_3 = -T_{L_t} \boldsymbol{q}_t + m_l g \boldsymbol{e}_3 \\ m_{Q_i} \ddot{\boldsymbol{x}}_{Q_i} = T_i \boldsymbol{q}_i - T_{Q_i} \boldsymbol{r}_{Q_i} + m_{Q_i} g \boldsymbol{e}_3 \end{cases} \tag{2}$$

where $\ddot{\boldsymbol{x}}_l$ denotes the load linear acceleration, $\ddot{\boldsymbol{x}}_{Q_i}$ the acceleration of quadrotor $i$, $g$ the gravitational acceleration, $T_i$ the tension applied by link $i$, $T_{Q_i}$ the thrust applied by quadrotor $i$, and $\boldsymbol{r}_{Q_i}$ the direction of that thrust, which coincides with the z-axis of the vehicle. The scalar $T_{L_t}$ and unit vector $\boldsymbol{q}_t \in \mathbb{S}^2$ define the total tension norm and direction, respectively, and are such that $T_{L_t} \boldsymbol{q}_t = T_1 \boldsymbol{q}_1 + T_2 \boldsymbol{q}_2$.

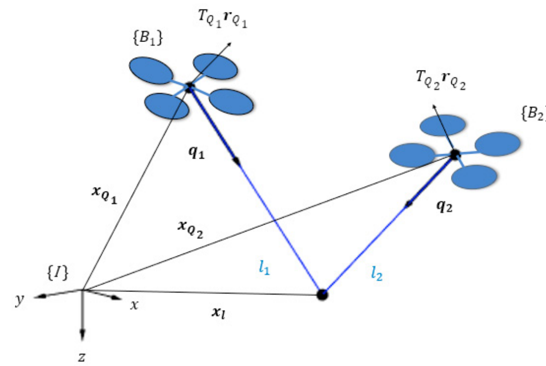


Fig. 1. Illustration of the problem statement (n = 2)

For the sake of simplicity, we assume that, for each quadrotor, an inner loop controller provides tracking of angular velocity commands and consider only the kinematics of $\boldsymbol{r}_{Q_i}$, assuming that the extra angular degree of freedom is independently controlled.

Using (1) and (2) similarly to Lee et al. (2013), and performing additional algebraic manipulations, the complete model can be written as

$$\begin{cases} \dot{\boldsymbol{x}}_l = \boldsymbol{v}_l \\ \dot{\boldsymbol{v}}_l = g\boldsymbol{e}_3 - M_q^{-1} \displaystyle\sum_{i=1}^{n} \alpha_i \boldsymbol{q}_i \\ \ddot{\boldsymbol{q}}_i = -\|\dot{\boldsymbol{q}}_i\|^2 \boldsymbol{q}_i + \dfrac{1}{l_i} \Pi_{q_i} \Big( \dfrac{1}{m_{Q_i}} T_{Q_i} \boldsymbol{r}_{Q_i} - M_q^{-1} \displaystyle\sum_{j=1}^{n} \alpha_j \boldsymbol{q}_j \Big) \\ \dot{\boldsymbol{r}}_{Q_i} = \boldsymbol{u}_{Q_i} \end{cases} \tag{3}$$

where the following variables are introduced

- $\boldsymbol{v}_l$ is the velocity of the load;
- $M_q$ is the positive definite symmetric matrix given by $M_q = m_l I + \sum_{i=1}^{n} m_{Q_i} \boldsymbol{q}_i \boldsymbol{q}_i^T$
- $\alpha_i$ is an auxiliary variable given by $\alpha_i = \boldsymbol{q}_i^T T_{Q_i} \boldsymbol{r}_{Q_i} + m_{Q_i} l_i \|\dot{\boldsymbol{q}}_i\|^2$
- $\boldsymbol{u}_{Q_i}$ is the simplified quadrotor angular velocity input, which satisfies $\boldsymbol{r}_{Q_i}^T \boldsymbol{u}_{Q_i} = 0$.

One aspect to consider when analysing the model equations is the fact for each quadrotor only the thrust component that is parallel to the respective links $\boldsymbol{q}_i^T T_{Q_i} \boldsymbol{r}_{Q_i}$ has an effect on the dynamics of the load, whereas the perpendicular component of the thrust $\Pi_{\boldsymbol{q}_i} T_{Q_i} \boldsymbol{r}_{Q_i}$ can be used to control the direction of cable $i$. This already gives insight to the control strategy that will be developed in the following sections.