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Surface wave in a Maxwell liquid-saturated poroelastic layer

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ABSTRACT

An analytical approach of the propagation and attenuation of Love waves in a viscoelastic liquid-saturated poroelastic layer has been considered in this paper. The equations of motion have been formulated separately for different media under suitable boundary conditions at the interface of viscoelastic liquid, poroelastic layer and elastic substrate. Following Biot's theory of poroelasticity, a new accurate and simple generalized dispersion equation has been established to design Love wave liquid sensors. The effect of liquid shear viscosity on the Love waves velocity has been studied. The influence of thickness and porosity of the waveguide layer has also been shown on the Love waves velocity and attenuation. The various investigations results can serve as benchmark solutions in design of liquid sensors and nondestructive testing.

1. Introduction

Surface acoustic wave have been successfully applied in many fields including material characterization, seismology and nondestructive testing. More recently, surface acoustic wave based sensors for applications to liquids environments have been extensively investigated [1–12]. The surface acoustic wave sensor directly contacts the liquid to be tested to characterize its physical properties.

In surface acoustic wave devices, the acoustic wave travels at the surface of the propagating medium and its energy is confined within one wavelength of depth; it follows that the surface acoustic wave properties (velocity and attenuation) are highly affected by any physical changes that occur at the surface of the propagating medium, when the surface acoustic wave device interacts with an external environmental stimuli. In the presence of a liquid environment, the wave properties can be perturbed by the changes in the physical properties of the liquid contacting the sensor surface.

The design of liquid sensors requires the selection of several parameters, such as the acoustic wave polarizations and the wave-guiding medium (i.e. finite thickness, homogeneous or non-homogeneous). This paper proposes a novel generalized dispersion equation to design Love wave liquid sensors, and intends the attention of the researchers to viscoelasticity of liquids and poroelasticity of layer that strongly affect the Love wave behavior.

2. Theoretical analysis

Love waves are a type of surface acoustic wave characterized by one shear horizontal particle mechanical displacement component u_2 dominant over the vertical and longitudinal ones. The Love wave liquid sensor consists of piezoelectric substrate, guiding layer and sensitive liquid. Due to the in-plane polarization, the Love waves are suitable for travel at a surface contacting a liquid environment. The number of Love modes that can propagate in the layer/substrate medium depends on the layer thickness, but the essential condition for the propagation of the Love waves is that the shear bulk wave velocity of the substrate is larger than the shear bulk wave velocity of the layer. Higher order Love modes develop at their respective cut-off frequencies, which are related to the thickness of the layer: they are dispersive as their velocity depends on the layer thickness, rather than on the substrate and the layer's material properties. Therefore, to describe the waveguide structure that guides Love waves, we consider a three-layer system consisting of a viscoelastic liquid, a poroelastic layer and an elastic substrate, see Fig. 1. The finite substrate occupies the positive space $x_3 > 0$. The layer rests on top of the substrate and has thickness h. On top of that rests the viscoelastic liquid which occupies the negative half-space $x_3 < -h$.

2.1. Guiding poroelastic layer and elastic substrate

To get a solution for Love wave propagating in the poroelastic layer, Biot's theory is used [13,14]. The poroelastic layer is composed of a solid skeleton and pore space. Moreover, the Love wave

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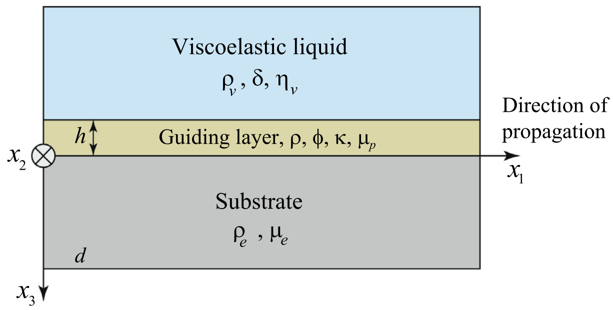

Fig. 1. The schematic representation of the Love wave. x_1: propagation direction of Love wave. x_2: polarization direction of Love wave. ρ_v, δ and η_v describe, respectively, the density, relaxation time and dynamic viscosity of the viscoelastic liquid. For the guiding poroelastic layer, ρ is the mixture density and the layer porosity and permeability are represented respectively by ϕ and κ; μ_p is the shear modulus of the porous framework. ρ_e and μ_e correspond to the density and shear modulus of the elastic substrate.

is taken to propagate in the x_1-direction, with shear displacement in the x_2-direction. A plane wave in the x_1-direction is considered, with displacement in x_2-direction only, $\mathbf{u}^{(p)} = \left(0, u_2^{(p)}, 0\right)$. Owing to symmetry, the displacement should be independent of $x_2, u_2^{(p)} = u_2^{(p)}(x_1, x_3)$. Therefore, the equation that governs the particle motion in a guiding layer saturated with a liquid is given by [15]

$$\left(b + m\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}\right)u_2^{(p)} - \frac{\rho}{\mu_p}\left[b + \left(m - \frac{\rho_f^2}{\rho}\right)\frac{\partial}{\partial t}\right]\frac{\partial^2 u_2^{(p)}}{\partial t^2} = 0 \tag{1}$$

where $b = \eta_f/\kappa$ and $m = \rho_f\tau/\phi$ represent respectively, the viscous and inertial coupling between the solid and fluid phases of the poroelastic layer [13]. Thus, the layer porosity and permeability are represented respectively by ϕ and κ; η_f describes the dynamic viscosity of the fluid phase and τ is the pores tortuosity. The parameter $\rho = \phi\rho_f + (1-\phi)\rho_s$ is the mixture density whereas ρ_f and ρ_s denote the densities of the solid and fluid phases of the poroelastic layer. In addition, μ_p is the shear modulus of the porous framework. Thus, in this work the substrate is considered to be a finite medium and the particle displacement $u_2^{(e)}$ is governed, using the elastodynamic theory by the Navier's equation

$$\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}\right)u_2^{(e)} - \frac{1}{c_e^2}\frac{\partial^2 u_2^{(e)}}{\partial t^2} = 0 \tag{2}$$

where $c_e = \sqrt{\mu_e/\rho_e}$ is the bulk shear wave velocity in the substrate.

2.2. Viscoelastic liquid

To describe the viscoelasticity of the liquid, the Maxwell model which introduces a viscoelastic response of liquids at high frequencies is employed. The model consists of a spring and a damper connected in series. The damper represents energy losses and is characterized by the viscosity η_v, whereas the spring represents the energy storage and is characterized by the elastic shear modulus μ. These two quantities are related through the relaxation time δ, which is the characteristic time for the transition between viscous and elastic behavior $\delta = \eta_v/\mu$ (see for example [16]). Thus, the liquid motion is only produced by wave propagation in the poroelastic layer. Furthermore, since only shear deformation arises during transverse waves propagation, we can also ignore the pressure gradient. In addition, the linearized Navier-Stokes equation that governs the viscoelastic liquid motion can be simplified to the following

$$\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}\right)v_2 - \frac{\rho_v}{\eta_v}\left(1 + \delta\frac{\partial}{\partial t}\right)\frac{\partial v_2}{\partial t} = 0 \tag{3}$$

where v_2 is the velocity component along the x_2-direction and ρ_v is the liquid density.

2.3. General solution of wave equations

For a plane harmonic wave propagation, the solution of Eqs. (1)–(3) are sought in the form

$$\begin{Bmatrix} v_2 \\ u_2^{(p)} \\ u_2^{(e)} \end{Bmatrix}(x_1, x_3) = \begin{Bmatrix} V_2(x_3) \\ U_2^{(p)}(x_3) \\ U_2^{(e)}(x_3) \end{Bmatrix} e^{j(kx_1 - \omega t)} \tag{4}$$

where ω is the angular frequency. Love wave propagating in the poroelastic layer undergoes attenuation, hence, the wavenumber k along the propagation direction of the Love wave becomes complex, $k = k_0 + j\alpha$, the real part k_0 describes the Love wave velocity, the imaginary part α, is the Love wave attenuation induced by the viscoelastic liquid. After substitution of Eq. (4) into Eqs. (1)–(3), the x_3 dependence can be expressed as

$$U_2^{(p)}(x_3) = A_p\cos\left(\beta_p x_3\right) + B_p\sin\left(\beta_p x_3\right),$$
$$U_2^{(e)}(x_3) = A_e e^{-\beta_e x_3} + B_e e^{\beta_e x_3},\ V_2(x_3) = A_v e^{\beta_v x_3}$$

where A_p, B_p, A_e, B_e and A_v are arbitrary amplitudes and the wavenumbers β_p, β_e and β_v are given in the following form

$$\beta_p = \sqrt{\frac{\omega^2}{\mu_p}\left(\rho - \frac{\rho_f^2}{m + \frac{jb}{\omega}}\right) - k^2},\ \beta_e = \sqrt{k^2 - \frac{\omega^2}{c_e^2}},$$
$$\beta_v = \sqrt{k^2 - \frac{j\omega\rho_v}{\eta_v}(1 - j\omega\delta)}$$

In addition, the shear stress components that will be used in boundary and interface conditions are given by

$$\begin{Bmatrix} \sigma_{23} \\ \sigma_{23}^{(p)} \\ \sigma_{23}^{(e)} \end{Bmatrix} = \begin{Bmatrix} \frac{\eta_v}{1-j\omega\delta}\partial_{x_3} v_2 \\ \mu_p\partial_{x_3} u_2^{(p)} \\ \mu_e\partial_{x_3} u_2^{(e)} \end{Bmatrix} = \begin{Bmatrix} \frac{\eta_v}{1-j\omega\delta}\Sigma_v(x_3) \\ \mu_p\Sigma_p(x_3) \\ \mu_e\Sigma_e(x_3) \end{Bmatrix} e^{j(kx_1 - \omega t)} \tag{5}$$

where the x_3 dependence is defined as

$$\Sigma_p(x_3) = \beta_p\left[B_p\cos\left(\beta_p x_3\right) - A_p\sin\left(\beta_p x_3\right)\right],$$
$$\Sigma_e(x_3) = \beta_e\left(B_e e^{\beta_e x_3} - A_e e^{-\beta_e x_3}\right),\ \Sigma_v(x_3) = A_v\beta_v e^{\beta_v x_3}$$

2.4. Boundary conditions and dispersion relation

The following boundary conditions: continuity of shear stress and velocity (or displacement), and traction-free outer surface are suitable

$$v_2 + j\omega u_2^{(p)}|_{y=-h} = 0,\ \sigma_{23}^{(v)} - \sigma_{23}^{(p)}|_{y=-h} = 0,$$
$$u_2^{(p)} - u_2^{(e)}|_{x_3=0} = 0,\ \sigma_{23}^{(p)} - \sigma_{23}^{(e)}|_{x_3=0} = 0,\ \sigma_{23}^{(e)}|_{x_3=d} = 0$$

According to the above boundary conditions, the resolution of Eqs. (1)–(3) leads to the following equation:

$$\left[\mu_p^2\beta_p^2 + \frac{j\omega\eta_v\beta_v}{1 - j\omega\delta}\mu_e\beta_e\tanh\left(\beta_e d\right)\right]\sin\left(\beta_p h\right)$$
$$+\mu_p\beta_p\left[\frac{j\omega\eta_v\beta_v}{1 - j\omega\delta} - \mu_e\beta_e\tanh\left(\beta_e d\right)\right]\cos\left(\beta_p h\right) = 0 \tag{6}$$

Eq. (6) represents the implicit dispersion equation of Love waves propagating in viscoelastic liquid-saturated poroelastic layer. To solve the Eq. (6) Mathematica Software is used to find the real