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Pricing and hedging foreign equity options under Hawkes jump–diffusion processes

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A B S T R A C T

In this paper, we propose a valuation of foreign equity options using a Hawkes jump–diffusion model that allows for clustered jumps as well as cross-market jump propagation. We derive the semi-analytical valuation formulae for these options using Fourier transform method. The Greeks and the optimal option hedging strategies under mean–variance criterion are also given. We find that Hawkes jump–diffusion model produces heavier tailed distributions with higher peaks than Poisson jump–diffusion model, which accordingly results in higher option prices under Hawkes model for deep out-of-the-money options.

1. Introduction

With the globalization of financial markets and the liberalization of investment regulations around the world over many decades, there have been significantly large amounts of cross-border capital flows into the international financial markets. For example, while gross cross-border portfolio (equity and bond) flows for the U.S. were equivalent to only 4% of GDP in 1975, this share increased to 100% in the early 1990s, and further increased to 245% by 2000 [1]. Such a massive increase in the amounts of capital flows has caused fluctuations in foreign exchange rates as well as foreign asset prices. Therefore, this phenomenon has drawn interest from investors, policy makers and scholars. Unlike a single risk factor, two risk factors (exchange rate and equity price) sometimes interplay with each other, which may lead to exaggeration of total risk (see, e.g., Ajayi and Mougoué [2]; Phylaktis and Ravazzolo [3]; Chen et al. [4]). To manage such multidimensional risks, foreign equity options are created as important and efficient instruments. Therefore, foreign equity options are gradually becoming popular over-the-counter (OTC) derivatives among industry practitioners and have attracted the research interest of academic community.

In general, the value of a foreign equity option depends on the value of the underlying foreign asset as well as the exchange rate. Based on the currency in which the strike price is denominated, a foreign equity option can be divided into two types: (i) an option with strike price denominated in the foreign currency (FEO_f) and (ii) an option with strike price denominated in domestic currency (FEO_d). Pricing a foreign equity option is more complicated than pricing a plain vanilla option because the former has a much more complex structure. In the extant literature, foreign equity option pricing is first studied under the Black–Scholes framework [5]. There are other studies that use some variants of Black–Scholes model, such as stochastic correlation and volatility models (see, e.g., Wong and Lo [6]; Ma [7]; Xu et al. [8]).

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In a pioneering study, Huang and Hung [9] use a Lévy process to price the foreign equity option where the model allows jumps, which are considered to be an essential feature of the foreign asset price and the exchange rate processes. Subsequent Lévy models take into account stochastic volatility or change the distribution of the jump size (see, e.g., Qi and Xu [10]; Gong and Zhuang [11]; Cui et al. [12]). In addition, some other pricing frameworks are also proposed in the literature. For instance, Xu et al. [13] model the underlying foreign asset price and the exchange rate via affine jump–diffusion processes with stochastic volatility. Others like Fan et al. [14] employ Markovian regime-switching mean-reversion model whereas Ulyah et al. [15] utilize a bivariate Bernoulli and bivariate asymmetric Laplace jump–diffusion model to price short-dated foreign equity options.

The Lévy process could better describe the fluctuations of the foreign asset price and the exchange rate than the Brownian motion in that the former allows for the occurrence of jumps. However, Lévy processes cannot explain the contagious links between different markets or different assets [16]. This limitation has an important implication in pricing foreign equity options. Due to large and growing amounts of cross-border capital flows, the foreign exchange rates and foreign equity prices are expected to affect each other. For example, Baily and Chung [17] find that foreign exchange has an impact on equity prices for Mexico. To explain such interrelationships between the foreign exchange and equity prices, there are theoretical papers that model such relationships. For example, Hua and Rey [1] develop an equilibrium model where exchange rate, stock prices and capital flows are jointly determined. They find that the model predictions are strongly supported by empirical data from 17 OECD countries. Similarly, Pavlova and Rigobon [18] present a model with a rich set of implications on how stock, bond and foreign exchange market co-move. Therefore, it is likely that a jump in one market may cause jump in another. In other words, a shock in one foreign exchange market may lead to a shock in the foreign equity market and vice versa.

In addition to the interrelationship between the foreign exchange and foreign equity markets, each market is characterized by jump clustering. The phenomenon of jump clustering in a single financial market are regularly observed (see, e.g., Maheu and McCurdy [19]; Aït-Sahalia et al. [16]).[1] To account for these two phenomenons (i.e., interrelationship and jump clustering), mutually exciting Hawkes processes are used in this study, which are originally proposed by Hawkes [21] and have the characteristics of self-excitement and cross-excitement, to model the correlated jump events in the foreign asset price and the exchange rate. The Hawkes processes differ from the usual point processes, such as Poisson processes, in that jumps in Hawkes processes are dependent rather than mutually independent. In Hawkes processes, the occurrence of a jump in one price increases the probability of jumps in its own price as well as the prices of other assets. Therefore, Hawkes processes allows for both jump clustering and financial contagion.

The Hawkes processes have been extensively applied in finance research (see, e.g., Bacry and cois Muzy [22]; Ma and Xu [23]; Ma et al. [20]; Hawkes [24]). In particular, Rambaldi et al. [25] has used Hawkes process to model foreign exchange market activity. However, to the best of our knowledge, Hawkes processes have not been applied to the pricing of foreign equity options. Therefore, the main contribution of this study is the use of Hawkes jump–diffusion processes to model the interrelated dynamics of the underlying foreign asset price and the exchange rate. We adopt the generalized Fourier inverse transform to derive the semi-analytical pricing formulae for the foreign equity options.

This study contributes to the existing literature in several ways. First, we propose a more general model to describe the cross-correlated jumps in the foreign equity price and foreign exchange rate as well as the clustered jumps in the individual market. Second, we derive the semi-analytical pricing formulae for foreign equity options. Third, we obtain the Greeks, and especially the optimal dynamic option hedging strategies under the mean–variance criterion. Fourth, the implications of the clustered jumps for valuing foreign equity options are revealed.

The remainder of this paper is organized as follows. In Section 2, we introduce mutually exciting Hawkes processes, Hawkes jump–diffusion processes, and derive the joint characteristic function of Hawkes jump–diffusion processes under the general affine jump–diffusion framework. Then, we apply Hawkes jump–diffusion processes to model the dynamics of the underlying foreign asset price and the exchange rate. In Section 3, we derive the pricing formulae for foreign equity options using the Fourier transform method. Section 4 presents the optimal portfolio processes for hedging these options, as well as some Greeks such as the delta, theta and gamma. In Section 5, we use numerical examples to discuss the sensitivity of the option prices to the parameters of Hawkes jump processes, and to compare the option prices obtained from the proposed model and Poisson jump–diffusion model. Finally, we conclude this paper.

2. Model

Consider a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{Q})$, where $\{\mathcal{F}_t\}$ is a right-continuous and complete filtration representing the available information flow and $\mathbb{Q}$ is a risk-neutral probability measure. Assume that all the stochastic processes in this paper are adapted to $\{\mathcal{F}_t\}$. Denote by $\mathbb{E}$ the expectation under the probability measure $\mathbb{Q}$.

[1] The reason for jump clustering can be explained as follows. When news initially arrives, we expect further news to arrive that would clarify the initial news, or that would reveal the extent of its impact. In this case, security prices would respond not only to the initial news, but also to the ways in which market participants and firms react to the news. Therefore, theoretically, we expect to observe jump clustering [20].