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Trade duration risk in subdiffusive financial models*

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Abstract

Subdiffusive processes are employed in finance to explicitly accommodate in return models the presence of random waiting times between price innovations, often referred to as "trade duration". In this paper we argue that pricing models based on subdiffusions naturally account for the presence of a trade duration market price of risk. In particular we make a case for *tempered subdiffusive* models, which are able to capture the time multiscale properties of equity prices, that is, the fact that different return idleness patterns are shown at different time scales. We explain the role in duration risk pricing of the stability and tempering parameters of a tempered subdiffusion, and show that option valuation can be performed using standard integral representations.

Keywords: Duration risk, subdiffusions, tempered subdiffusions, derivative pricing, inverse tempered stable subordinator, Lévy processes.
JEL classification: C65, G13

1 Introduction

Subdiffusive stochastic processes are used in science to model natural phenomena of slow particle displacement, typically in fluid-dynamics, hydrology, engineering and physics. Such processes allow explicit modelling of resting times between particle movements, that in financial asset models can be interpreted as extended random periods of zero returns between price revisions, that is, the presence of a trade duration (Dufour and Engle, 2000).

Transition densities of subdiffusive models are characterized as solutions of Fokker-Planck equations of fractional order, and their stochastic representations are typically given in the form of Lévy processes time-changed with an inverse-stable subordinator (Saichev and Zaslavsky 1997, Baeumer and Meerschaert 2001, Meerschaert et al. 2002). When the inverse subordinator is instead specified by a *tempered* (that is, with exponentially dampened jumps) stable process the resulting process is denominated a tempered subdiffusion.

Subdiffusions have been introduced in mathematical finance in connections to continuous time random walks by the pioneering work of Scalas et al. (2000), where price densities are

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