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Frequency domain bootstrap for ratio statistics under long-range dependence

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A B S T R A C T

A frequency domain bootstrap (FDB) is a common technique to apply Efron's independent and identically distributed resampling technique (Efron, 1979) to periodogram ordinates – especially normalized periodogram ordinates – by using spectral density estimates. The FDB method is applicable to several classes of statistics, such as estimators of the normalized spectral mean, the autocorrelation (but not autocovariance), the normalized spectral density function, and Whittle parameters. While this FDB method has been extensively studied with respect to short-range dependent time processes, there is a dearth of research on its use with long-range dependent time processes. Therefore, we propose an FDB methodology for ratio statistics under long-range dependence, using semi- and nonparametric spectral density estimates as a normalizing factor. It is shown that the FDB approximation allows for valid distribution estimation for a broad class of stationary, long-range (or short-range) dependent linear processes, without any stringent assumptions on the distribution of the underlying process. The results of a large simulation study show that the FDB approximation using a semi- or nonparametric spectral density estimator is often robust for various values of a long-memory parameter reflecting magnitude of dependence. We apply the proposed procedure to two data examples.

1. Introduction

The bootstrap method known as Efron's independent and identically distributed (iid) bootstrap (individual data resampling) (Efron, 1979) is a powerful tool for approximating certain statistical properties, such as variance, bias, or distribution. In particular, it has been mainly leveraged for statistics whose analytic forms of certain properties cannot be easily obtained without expending excessive calculation efforts.

In time series analysis, Singh (1981) reports that under short-range dependence (SRD), Efron's iid bootstrap could be invalid. Therefore, to address this issue with respect to the time domain, Carlstein (1986) suggests the nonoverlapping block bootstrap, Künsch (1989) proposes the moving block bootstrap, and Bühlmann (1997) puts forward the autoregressive (AR) sieve bootstrap. Additionally, the recent extension of bootstrap methods under SRD has proceeded to long-range dependent (LRDt) time processes, even though Lahiri (1993) shows that the moving block bootstrap could be invalid in approximating the sample means for a class of LRDt time series generated by transformations of Gaussian processes. Kim and Nordman (2011) investigated the properties of bias, variance, and distribution of sample means using moving and nonoverlapping block bootstrap for stationary linear LRDt processes; they considered the optimal block choice, based

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on the large sample mean squared error of a bootstrap variance estimator. The AR sieve bootstrap under long-range dependence (LRD) has been justified for causal linear LRDt time processes under certain conditions of the theorem, as presented in Kapetanios and Psaradakis (2006); additionally, Bühlmann (1997), Kapetanios and Psaradakis (2006), and Poskitt (2008) each investigated the optimal order selection method for the sieve bootstrap under SRD or LRD.

In comparison to time domain bootstrap methods, a frequency domain bootstrap (FDB) method involves resampling periodogram ordinates that are studentized by spectral density estimates under weak dependence (Dahlhaus & Janas, 1996; Franke & Härdle, 1992). Franke and Härdle (1992) developed the FDB using kernel-based spectral density estimates and investigated their consistency. However, the method works only under a certain class of statistics, such as *ratio statistics*—normalized spectral means, autocorrelation estimates, normalized spectral density function estimates, and estimates of Whittle parameters (Dahlhaus & Janas, 1996). The conventional nonparametric spectral density estimation (NSDE) for the FDB under weak dependence uses a kernel function to smooth periodogram ordinates. For stationary and linear LRDt time processes, Kim and Nordman (2013) investigated an FDB for approximating the distribution of Whittle estimators, by using parametric spectral density estimates under LRD.

Semiparametric approaches to estimating spectral density by using fractional exponential and fractional AR models have been suggested by Bhansali, Giraitis, and Kokoszka (2006), Hurvich and Brodsky (2001), Moulines and Soulier (1999, 2000), and Narukawa and Matsuda (2011). Those methods use the log-periodogram regression approach to combine the long-memory term, and the AR-approximate parametric term of a short-memory part of the spectral density function of fractional exponential models. Recently, Kim, Lahiri, and Nordman (2018) examined the NSDE under LRD, based on the smoothness of the periodogram by a kernel function; they provide optimal kernel bandwidths based on uniform and pointwise concepts.

Our goal in the current study is to establish the FDB inference about ratio statistics for a different but practically wide class of stationary linear processes exhibiting strong dependence; these include popular stationary linear LRDt models such as fractional Gaussian processes (FGN; Mandelbrot & Van Ness, 1968) and the fractional autoregressive integrated moving average (FARIMA; Adenstedt, 1974; Granger & Joyeux, 1980; Hosking, 1981). In the current study, we present the FDB method, to be consistent with distribution estimation under mild and flexible conditions entailing strong dependence.

The remainder of this paper is organized as follows. In Section 2, we describe the FDB method under LRD and show the validity of FDB inference for ratio statistics under LRD. We investigate in Section 3 numerical studies while considering two spectral density estimation techniques, such as semi- and nonparametric approaches. In Section 4, we provide data examples to estimate confidence intervals, and finally we provide concluding remarks in Section 5. Proofs for theorems, and some simulation results, are found in Appendix A.

2. Frequency domain bootstrap under long-range dependence

2.1. Target process

Suppose that $\{X_t\}_{t\in\mathbb{Z}}$ is a real-valued, stationary linear process defined as

$$X_t = \mu + \sum_{j\in\mathbb{Z}} b_j \varepsilon_{t-j}, \quad t \in \mathbb{Z}, \tag{1}$$

where $\mu = \mathrm{E}X_t$ and $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is an iid sequence with $\mathrm{E}\varepsilon_t = 0$, $\mathrm{E}\varepsilon_t^2 = \sigma_\varepsilon^2 \in (0, \infty)$ and $\mathrm{E}\varepsilon_t^4 < \infty$. The sequence of constants, $\{b_t\}_{t\in\mathbb{Z}} \subset \mathbb{R}$, satisfies $\sum_{t\in\mathbb{Z}} b_t^2 < \infty$ with $b_0 = 1$. The spectral density function of the process $\{X_t\}_{t\in\mathbb{Z}}$ is defined as

$$f(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi} |b(\lambda)|^2 \quad \lambda \in \Pi \equiv (-\pi, \pi],$$

where $b(\lambda) = \sum_{j\in\mathbb{Z}} b_j \exp(\imath j\lambda)$ and $\imath = \sqrt{-1}$. In addition, the spectral density function, $f(\cdot)$, has the common characteristics of an LRDt process, including

$$f(\lambda) \sim C_f |\lambda|^{-(1-\theta)} \quad \text{as} \quad \lambda \downarrow 0, \tag{2}$$

with a long-memory parameter $\theta \in (0, 1)$ and a constant $C_f \equiv C_f(\theta) > 0$. Here, "$\sim$" denotes that the ratio of quantities on the left and right-hand sides of (2) is 1 at the limit. If $\theta = 1$, the process is a short-memory one. An alternative formulation for showing the properties of long memory is a covariance function of the process $r(k) \equiv \mathrm{Cov}(X_0, X_k) = \int_\Pi e^{-\imath k\lambda} f(\lambda) d\lambda$, $k \in \mathbb{Z}$, which satisfies a slow decay condition as

$$r(k) \sim C_r k^{-\theta} \quad \text{as} \quad k \to \infty, \tag{3}$$

for $\theta \in (0, 1)$ and some constant $C_r \equiv C_r(\theta) > 0$, whereby the partial covariance summation, $\sum_{k=1}^n r(k) \propto n^{1-\theta}$, diverges as $n \to \infty$. Characterizations of long memory through the properties of covariance (3) or the spectral density (2) are related (see Beran, 1994; Robinson, 1995a).

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