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FK–Ising coupling applied to near-critical planar models

Federico Camia[a,b], Jianping Jiang[c,*], Charles M. Newman[d,c]

[a] *Division of Science, NYU Abu Dhabi, Saadiyat Island, Abu Dhabi, UAE*
[b] *Department of Mathematics, VU Amsterdam, De Boelelaan 1081a, 1081 HV Amsterdam, The Netherlands*
[c] *NYU-ECNU Institute of Mathematical Sciences at NYU Shanghai, 3663 Zhongshan Road North, Shanghai 200062, China*
[d] *Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, NY 10012, USA*

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Abstract

We consider the Ising model at its critical temperature with external magnetic field $ha^{15/8}$ on $a\mathbb{Z}^2$. We give a purely probabilistic proof, using FK methods rather than reflection positivity, that for $a = 1$, the correlation length is $\geq$ *const*. $h^{-8/15}$ as $h \downarrow 0$. We extend to the $a \downarrow 0$ continuum limit the FK–Ising coupling for all $h > 0$, and obtain tail estimates for the largest renormalized cluster area in a finite domain as well as an upper bound with exponent $1/8$ for the one-arm event. Finally, we show that for $a = 1$, the average magnetization, $\mathcal{M}(h)$, in $\mathbb{Z}^2$ satisfies $\mathcal{M}(h)/h^{1/15} \to$ some $B \in (0, \infty)$ as $h \downarrow 0$.
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1. Introduction

1.1. Overview

In a recent paper [6], the authors obtained upper and lower bounds of the form $C_0 H^{8/15}$ and $B_0 H^{8/15}$ as $H \downarrow 0$, for the exponential decay rate (the mass gap or inverse correlation length) of the (β_c, H) planar $(\mathbb{Z}^2)$ Ising model at critical inverse temperature β_c with magnetic field $H \geq 0$. The lower bound derivation used methods based on the FK random cluster representation of the Ising model, including the use of the Radon–Nikodym derivative of the distribution of the model with external field with respect to the distribution of the model without external field. This

* Corresponding author.
E-mail addresses: federico.camia@nyu.edu (F. Camia), jjiang@nyu.edu (J. Jiang), newman@cims.nyu.edu (C.M. Newman).

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derivative appears implicitly in [7] (see also [ref. 16 of [7]]), as we learned after the first version of this paper was posted in 2017. The upper bound, on the other hand, was derived in [6] by quite different methods based on reflection positivity.

Here we extend the FK methods of [6] in several ways. First we give (in Theorem 1 and Corollary 1) an alternative derivation of the $H^{8/15}$ upper bound using only FK-based methods. Then in Theorem 2 we show that the FK/Ising coupling of [6] is valid for the scaling limit continuum FK measure ensemble with positive renormalized magnetic field h, extending the continuum Edwards–Sokal type coupling shown in [2] beyond the $h = 0$ case. This coupling is then applied to obtain in Theorem 3 for $h \geq 0$ precise tail behavior (of the form $\exp(-Cx^{16})$) for the largest total mass in the ensemble of continuum FK measures in a bounded domain; this is analogous to the result of [14] for the tail of the largest cluster area in critical Bernoulli percolation. Tail behavior for both continuum and discrete FK models is derived in Sections 3 and 5 by using the FK/Ising coupling to relate moment generating functions for cluster size to those for Ising magnetization.

Our final main result (in Theorem 4) gives very precise behavior for the magnetization $\mathcal{M}(H)$ (expected spin value of the (β_c, H) Ising model on $\mathbb{Z}^2$) that improves the bounds from [3] that as $H \downarrow 0$

$$B_1 H^{1/15} \leq \mathcal{M}(H) \leq B_2 H^{1/15}. \tag{1}$$

The improved result is

$$\lim_{H\downarrow 0} \frac{\mathcal{M}(H)}{H^{1/15}} = B \in (0, \infty). \tag{2}$$

The derivation of (1) in [3] was fairly short, but the derivation of (2) in Section 1.2 is yet shorter and uses little more than the existence of a scaling limit magnetization field for $h \geq 0$ [4,5].

1.2. Main results

Let $a > 0$. Denote by P_h^a the infinite volume Ising measure at the inverse critical temperature β_c on $a\mathbb{Z}^2$ with external field $a^{15/8}h > 0$. Let $\langle\cdot\rangle_{a,h}$ be the expectation with respect to P_h^a. Let $\langle\sigma_x; \sigma_y\rangle_{a,h}$ be the truncated two-point function, i.e.,

$$\langle\sigma_x; \sigma_y\rangle_{a,h} := \langle\sigma_x\sigma_y\rangle_{a,h} - \langle\sigma_x\rangle_{a,h}\langle\sigma_y\rangle_{a,h}.$$

For $x, y \in \mathbb{R}^2$, let $|x - y| := \|x - y\|_2$ denote the Euclidean distance. Our first main result is:

Theorem 1. *There exist $C_2, C_3, C_4 \in (0, \infty)$ such that for any $a \in (0, 1]$, $h > 0$ with $a^{15/8}h \leq 1$, and $x, y \in a\mathbb{Z}^2$ with $|x - y| \geq C_2 h^{-8/15}$*

$$\langle\sigma_x; \sigma_y\rangle_{a,h} \geq C_3 a^{1/4} h^{2/15} e^{-C_4 h^{8/15}|x-y|}. \tag{3}$$

In particular, for a=1 and any $H \in (0, 1]$, we have for any $x', y' \in \mathbb{Z}^2$ with $|x' - y'| \geq C_2 H^{-8/15}$

$$\langle\sigma_{x'}; \sigma_{y'}\rangle_{1,H} \geq C_3 H^{2/15} e^{-C_4 H^{8/15}|x'-y'|}. \tag{4}$$

For $a = 1$, define the (lattice) mass (or inverse correlation length) $\tilde{M}(H)$ as the supremum of all $\tilde{m} > 0$ such that for some $C(\tilde{m}) < \infty$,

$$\langle\sigma_{x'}; \sigma_{y'}\rangle_{1,H} \leq C(\tilde{m}) e^{-\tilde{m}|x'-y'|} \text{ for any } x', y' \in \mathbb{Z}^2. \tag{5}$$

The following immediate corollary of Theorem 1 gives a one-sided bound for the behavior of $\tilde{M}(H)$ as $H \downarrow 0$, with the expected critical exponent $8/15$.

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