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An approach for hyperelastic model-building and parameters estimation a review of constitutive models

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ABSTRACT

This work presents a mathematical approach for the best way to structure hyperelastic models applicable to incompressible rubber-like materials; and it describes a model validation procedure using a step-by-step method for parameters estimation. This technique allows the validation of a restrained model as well as a complete one by doing it by graduation as the deformation increases. It builds on the restrained form (constrained model) essentially by progressively adding a higher degree term. The contribution of the added terms is irrelevant to the regime in which this constrained model provides a good fit to data. It becomes significant only where the deviation between the data and the restrain model prediction is important. It is a nonlinear process that leads to an optimal solution. After a concise appraisal of the underlying theoretical framework, the model-building strategies and parameters estimation method are presented. An attempt to understand and to elucidate how the existing attractive phenomenological models have been built is discussed. Furthermore, analytical and numerical results from hyperelastic modeling are compared using each of the two procedures first by deriving an optimal strain energy function which is then used in the formulation of a new constitutive model that generalizes the Hart-Smith model and second, by evaluating the correct and stable parameters values of rubbery materials taking into account the physical constraints that must be imposed on a realistic and physical model.

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1. Introduction

Physics investigations most often force researchers to adopt analytical models to examine or simulate phenomena. In hyperelastic modeling of rubber-like materials, a variety and a multitude of phenomenological constitutive models have been proposed either in strain-invariant-based form as that of Rivlin [55]; or stretch-based form as those of Valanis and Landel [66], Peng and Landel [53] and Ogden [50], Ogden [51]. The usual approach for building a phenomenological hyperelastic model is generally based on two principal points of view: the first one is based on assumptions, specific to the investigator, which consider a certain number of (first) terms of the (strain energy) Rivlin expansion; and the second takes the experimental established facts by Rivlin and Saunders [57] and by Treloar [62] as the basis for their modeling. Usually in the first case, a specific and explicit polynomial expression is assumed; examples include the earlier models of Mooney [46], Isihara et al. [31], Biderman (1958), Obata et al. [49], Tschoegl [64], James et al. [32,33] and more recent models [71,69,40,28,2]. In the second case, several theoretical or empirical models have been developed taking into account experimental established facts as constraints that must be imposed on a model [23,27,1,38,8,5]). The first approach generally leads to a *fixed-order problem*, then a linear problem. Subsequently to validating an assumed model, one ought to determine the parameters from fits of sets of experimental data in order to adjust the model

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predictions by minimizing errors. Frequently, the classical least squares procedure is applied. This procedure is based on fitting the coefficients of all of the terms of a model in a single least squares process. In most cases, several data sets of different deformation modes are simultaneously used for a better accuracy but without any physical constraints, i.e., without restrictions on sign of parameters [21,25,52,2,58,34,59]. In the opposite case, the number of terms used in building a model and its order are purely and directly established from fit of data. The analytical form (number of terms) of a model erected from the Rivlin series or from the Ogden expression (1984) is adapted to the complexity of the data (high-order or low-order materials). This second way leads to a *free-order nonlinear problem*, which remains now a difficult procedure. This is why investigators commonly assume a precise form involving a fixed number of terms of the strain energy as seen previously. Recently, an approach in stages [10,6], i.e., a continuous progression by stepped blocks, accessible enough to a non specialist in numerical computation, allows a simultaneous estimation of an optimal form of a model (free-order problem) and the corresponding parameters values by a multi-stage-identification process. Some investigators have chosen to do parameters identification by a step-by-step procedure [38] without constraints on parameters sign [28,7,43]. Others have imposed strict restrictions on parameters sign [26,11,15,8,5,9,14].

In the present works the two ways of building and assessing (density) energy function forms and the two techniques for estimating parameters will be investigated: direct and step-by-step procedures. At the end a new constitutive model would be suggested.

2. Procedures for parameters estimation: basic concepts

2.1. Classical procedure

The procedure most commonly used in numerical computation is the least squares method based on a single process of evaluating simultaneously all the parameters of a model. The use of least squares requires the user to provide an explicit form of the model (i.e. the number of known-order basic generating functions), which can be the order of polynomial approximation for linear least squares or the choice of a precise model (a fixed number of unknown-order monomial terms) for non-linear least squares. Note that in linear least squares method, the coefficients of all of the terms of a model are fitted in a single-step operation without any constraint on parameters sign [17,60,39,4]. For problems where the basic functions are unknown (as example: unknown coefficients and unknown exponents of monomials), it becomes a nonlinear problem. All usual nonlinear methods (e.g., Newton, Newton–Raphson, Gauss–Newton, Marquardt, Levenberg–Marquardt, etc.) being all iterative procedures, optimal use of nonlinear least squares requires good judgment and experience to establish good initial guesses to avoid local minima for stationary solution [39,36,37,42]. The convergence of these procedures depends on many factors: regularity of the Jacobian matrix, choice of the line search, etc. In general, no sign constraints exist on parameters values in nonlinear least squares method, except for methods as the so-called NNLS (non-negative least squares procedure) by Lawson and Hanson [39]. See also the works of Hartmann [26] and D'haes et al. [16].

2.2. Approach-in-stages procedure

This method is a different identification procedure done stage-by-stage enabling the weighting effects of each generating function to be felt in the relative partial identification domain [10]. This is contrary to the usual procedure that permits the evaluation of all of the terms of a model in a single process. The new approach makes it possible to estimate the real order of a function. It may be recalled that in the usual method, the investigator has to estimate this order by himself, based on his experience, knowledge etc. The approach-in-stages procedure graphically evaluates generating functions and parameters values step-by-step in a continuous multi-stage process.

2.2.1. Estimating the optimal order and the accurate base of approximation

A significant property of the approach-in-stages is its ability to evaluate an optimal approximation order, and to predetermine the correct number of basic functions generating the sough-after function [10]. As a result, the optimal model is precisely obtained [7,8].

For a better precision on parameters' values, the least squares method can be apply by taking as base of approximation the generating-functions obtained by the approach-in-stages procedure.

2.2.2. Inverse method

In linear plotting, the segment [0 1] represents a unity graduation in grid reference for the domain [0 1 0]. For functions study, one generally considers two principal subsets: [0 1], which is very small (in linear analysis consideration) and $[1\,\infty[$, which is extremely large.

- To invert the importance (the weight) of these two subsets, one has to take the inverse of the considered variable, so the segment [0 1] is converted into $[1\,\infty[$ and vice versa.
- To give the same influence to the two subsets, one considers the logarithmic function, thus, the segment [0 1] becomes $[-\infty\,0[$ and the interval $[1\,\infty[$ turns into $[0\,\infty[$.

This permits the extension of the study domain of the interval [0 1] to $[-\infty\,0[$ and $[1\,\infty[$ to $[0\,\infty[$ for better investigation and more precision. Thus, to begin by determining the lower order monomial when x tends towards zero, one displays the curve in bilogarithmic plot (Log x, Log $f(x)$); the slope of the linear part corresponds to the exponent of the monomial and the intercept to its coefficient. Note that for the monomials x^{β_j} with β_j a negative real number, i.e., $\beta_j < 0$, the equivalent function is obtained when x tends towards zero.

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