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A scoping inquiry into the potential contribution of Subjective Probability Theory, Dempster–Shafer Theory and Possibility Theory in accommodating degrees of belief in traveller behaviour research

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ABSTRACT

There is a small but growing interest in traveller behaviour research on investigating ways to identify and quantify degrees of belief (as subjective probabilities or other propositions) associated with behavioural responses, especially in the context of popular travel choice methods such as stated choice experiments, as a way of adding to our understanding of decision making in real-world contexts that are associated with inevitable risk and uncertainty. This paper reviews three major theories that are not well known in the transportation literature that have been developed in psychology and decision sciences to accommodate belief, namely Subjective Probability Theory, Dempster–Shafer Theory and Possibility Theory. We focus on how degrees of belief are measured in these theories. The key elements of each theoretical approach are compared, including their mathematical properties and evidence patterns. Despite their being few applications to date in transportation, the review promotes the relevance of accounting for degrees of belief in travel choice analysis.

Introduction

The economic environment is characterised by unmeasurable uncertainty rather than measurable risk (Knight, 1921). If a choice is made under risk, the probability distribution of all possible outcomes is known or can be calculated. Uncertainty is defined as "a quality depending on the amount, type, reliability and unanimity of information, and giving rise to one's degree of confidence in an estimate of relative likelihoods" (Ellsberg, 1961, p. 657), under which decision makers have to assess the probabilities of potential outcomes with some degree of vagueness, and rely on their beliefs to make the assessment. A person's confidence may vary with respect to different propositions, which are the objects of belief, i.e., sets of possible worlds or truth conditions (Huber, 2009). For example, she or he is more confident that the bus will arrive at the station on time than that it will be a rainy day tomorrow. The strength of confidence is measured by the degree of belief. Individuals use judgments of numerical probability to represent their degrees of beliefs, which are collected systematically and viewed as an approximation to the degrees of belief implicit in decision making (Idison et al., 2001). The degree of belief of a proposition is typically determined by evidence such as data information, and knowledge, which enable a decision maker to make a judgment and draw a conclusion (Kronprasert, 2012).

Given that belief plays a key role in decision making under uncertainty, it is useful to understand belief and to measure degrees of belief. A number of theories have been developed that focus on degrees of belief including Subjective Probability Theory (Ramsey, 1931; Savage, 1954), Dempster–Shafer Theory (Dempster, 1967, 1968; Shafer, 1976), and Possibility Theory (Zadeh, 1978; Dubois and Prade, 1988). The essential difference between the three theories can be best summarised as different mathematical properties (defined in detail in later sections but noting that A and B are any two variables or events or subsets of variables) that are used to account for degrees of belief. Under Subjective Probability Theory, degrees of belief are assumed to be additive (i.e., $\Pr(A) + \Pr(B) = \Pr(A \cup B)$ if $A \cap B = \varnothing$). Dempster–Shafer Theory treats degrees of belief as super-additive (i.e., $\text{Bel}(A) + \text{Bel}(B) \leqslant \text{Bel}(A \cup B)$); while Possibility Theory postulates degrees of belief to be sub-additive (i.e., $\Pi(A) + \Pi(B) \geqslant \max\{\Pi(A), \Pi(B)\} = \Pi(A \cup B)$). The aim of this paper is to provide an overview of these theories, with a focus on how the degree of belief is measured, and to promote the need to incorporate degrees of belief into studies of choice making behaviour in transportation. To date, such theories have attracted little attention by traveller behaviour researchers and it can be argued, given the accumulated evidence in psychology and decision sciences, that conditioning choice responses in methods such as stated choice experiments on the 'believability' of a hypothetical response reflecting real

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behaviour may offer a way of weighting such responses by some suitable metric representing the believability or confidence the analyst has in the evidence offered up by respondents in surveys. This is also a way of recognising and accounting for the confidence that the respondent has in their judgment and choice.

Subjective Probability Theory

The best developed account of degrees of belief is Subjective Probability Theory (Huber, 2012). The concept of subjective probability was originally proposed by Ramsey (1931) and further developed by Savage (1954). The entire theory of subjective probability is established around the notion of 'degree of belief' (Eriksson and Hájek, 2007). The operational explanation of subjective probability is "the probability of an uncertain event is the quantified measure of one's belief or confidence in the outcome, according to their state of knowledge at the time it is assessed" (Vick, 2002, p. 3). Subjective probabilities represent "degrees of belief in the truth of particular propositions", which reflect individuals' assessment based on their knowledge and opinions (Ayton and Wright, 1994, p. 164). Therefore, subjective probabilities actually represent the facts about a decision maker, not about the world, which arise as a response to the failure of frequency-based objective probability theory, when there is the occurrence of uncertain events (Pollock, 2006). Anscombe and Aumann (1963) use the horse race as a descriptive example of subjective probability, where individuals made bets according to their subjective probabilities of each horse winning with uncertain consequences. However, risky gambles, such as a roulette wheel, have a finite set of terminal outcomes associated with objective probabilities. Ferrell (1994, p. 413) concluded that "subjective probability can enter at any stage of the decision analysis process, implicitly and explicitly as a way of dealing with uncertainty ... as the means of quantifying the uncertainties in the models that relate the alternatives to possible consequences." Decision makers use "subjective probabilities to represent their beliefs about the likelihood of future events or their degree of confidence in the truth of uncertain propositions" (Brenner, 2003, p. 87). Consequently, to understand the nature of subjective probability can offer important insights into the structure of human knowledge and belief.

Subjective probabilities are constrained by axioms of classical probability theory and follow the laws of probability (Ayton and Wright, 1994). Under Subjective Probability Theory, degrees of belief (i.e., subjective probabilities) are additive (Huber, 2012). A probability space $(S, \mathcal{R}, \Pr)$ consists of a set S (i.e., the *sample space*), a σ-algebra $\mathcal{R}$ of subsets of S whose elements are called measurable sets, and a probability function $\Pr: \mathcal{R} \rightarrow [0,1]$, satisfying the following properties:

$$\Pr(X) \geqslant 0 \text{ for all } X \in \mathcal{R} \tag{1}$$

$$\Pr(S) = 1 \tag{2}$$

$$\Pr(X_1 \cup X_2 \cup \ldots \cup X_n \cup \ldots) = \Pr(X_1) + \Pr(X_2) + \ldots + \Pr(X_n) + \ldots, \text{ if the } X_n\text{'s are pairwise disjoint members of } \mathcal{R} \tag{3}$$

Property (3) is referred to as *countable additivity*, which can be simplified to *finite additivity* if $\mathcal{R}$ is a finite set:

$$\Pr(X_1 \cup X_2) = \Pr(X_1) + \Pr(X_2) \text{ if } X_1 \cap X_2 = \emptyset \tag{3'}$$

Ramsey (1931) proposed two ways to identify subjective probability: (i) *introspective interpretation*, i.e., measuring subjective probabilities by asking respondents; and (ii) *behaviourist interpretation*, i.e., defining subjective probabilities as a theoretical entity inferred from a choice. The behaviourist interpretation (i.e., subjective probabilities can be estimated from observed preference) was the dominant approach to the elicitation of subjective probabilities before the Ellsberg paradox (Ellsberg, 1961).

Subjective probabilities elicited from choice (i.e., the behaviourist interpretation) are always calculated based on a linear functional form (essentially all elements of influence are additive in the parameters and the attributes). So, coherent probabilities cannot be obtained, unless an individual's attitude toward uncertainty is neutral (Baron and Frisch, 1994). Given the limitation of the behaviourist interpretation, the introspective interpretation represents a more appealing way to measuring subjective probabilities. Since the 1980s, there have been an increasing number of studies in the area of psychology, behavioural and experimental economics, which directly asked respondents for their probability judgements over uncertain outcomes (see e.g., Kahneman et al., 1982; Heath and Tversky, 1991; Fox and Tversky, 1998; Wu and Gonzalez, 1999; Takahashi et al., 2007). For example, Heath and Tversky (1991) asked respondents to give probability assessments on football predictions and political predictions, and found that uncertainty has an impact on preference. In Wu and Gonzalez (1999), respondents were asked to provide their personal probability assessments on a number of events (e.g., national election and the number of University of Washington football team victories), and their judged probabilities were mapped into decision weights through the non-linear probability weighting function, which they referred to as a two-stage modelling process. Beach and Connolly (2005) defined the elicitation of subjective probability as "asking people to give a number to represent their opinion about the probability of an event".

Based on the behaviourist interpretation, Savage (1954) also suggested that the decision rule under uncertainty is to maximise expected utility based on assigned probabilities (i.e., Subjective Expected Utility Theory (SEUT)). In Savage's model, subjective probability and utility can be inferred simultaneously from observed preferences. For example, if there is no difference in a subject choosing: (1) winning \$10 if tomorrow rains and nothing if not, and (2) an expected win of \$5 (winning \$10 for a head when tossing a coin (with an objective probability of 0.5)), then we can infer a subjective probability of 0.5. The monetary value of the sure win can be varied so as to identify individuals' beliefs (subjective probabilities). This normative theory has no distinctive difference between risk and uncertainty, which also suggested that uncertainty may be equivalent to risk for a rational person. Ellsberg's two-colour example (see Appendix A for details), however, suggests that people are more willing to bet in the situation with known probabilities than without known probabilities. This typical behaviour is referred to as 'uncertainty or ambiguity aversion', which in turn highlights the important distinction between risk and uncertainty.

In a transportation context, Hensher et al. (2013) introduced subjective belief in a mixed multinomial logit choice model to identify *ex ante* support for specific road pricing schemes, such that the evidence in making a choice in a voting model is believable.[1] The approach is centred on a referendum voting choice model for alternative road pricing schemes in which they incorporated information that accounts for the degree of belief of the extent to which such schemes will make voters better or worse off. They capture the extent of deviation between an obtained belief probability and a per-

[1] We used the following scale: '*To what extent do you think that each of these schemes will make you better (or worse) off*'? (*0 = not at all, 100 = definitely*). Also, 'In answering this question, how well informed do you think you are that each of the schemes will make you better off'? 1–6 (1 = totally uninformed, 2 = strongly uninformed, 3 = moderately uninformed, 4 = moderately informed, 5 = strongly informed, 6 = totally informed); and 'In answering this question, how well informed do you think you are that each of the schemes will make you worse off'?