Travel Behaviour and Society 3 (2016) 29–37

Contents lists available at ScienceDirect

Travel Behaviour and Society

journal homepage: www.elsevier.com/locate/tbs

# A quantum utility model for route choice in transport systems

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ARTICLE INFO

*Article history:*
Received 11 April 2014
Received in revised form 3 July 2015
Accepted 8 July 2015
Available online 5 August 2015

ABSTRACT

One of the main components of the transport system is users' choice behaviour. Choices result from users' behaviour and are simulated by means of demand models. These models simulate how users' behaviour is influenced by system activities and supply performance. The most common demand approaches are based on the Random Utility Model (RUM).

According to the RUM, a user knows of and considers mutually exclusive alternatives and associates each alternative with a perceived utility. The choice probability for each alternative is estimated using the RUM. An analyst evaluates the same value of pre-trip choice probability in the case of a unique sequence of decisions for his final choice of an alternative as in the case of a not-unique sequence of decisions for his final choice of an alternative.

A new class of models simulates the case in which the user has an unclear sequence of decision for his final choice of an alternative. This model, the Quantum Utility Model (QUM), derives from quantum mechanics models. In QUM, it is possible to simulate the sequence of decisions in the cases of unique or not-unique pre-trip decision in the intermediate levels.

In this paper, a comparison between the RUM and the QUM for the transport demand simulation is reported. A specification of the model is reported for the route choice level. The models are specified and compared in terms of numerical results in two test networks.



## 1. Introduction

In transport systems theory, to simulate users' behaviour, the decision process is often based on Random Utility Models (RUMs), on the concept of rational users, and on the following assumptions (Von Neumann and Morgenstern, 1944; Ben-Akiva and Lerman, 1985; Cascetta 2009). The user considers mutually exclusive alternatives and adopts for each alternative a perceived utility function of a set of measurable characteristics. Perceived utility is not completely known by the analyst for the user; the analyst represents perceived utility with a random variable and evaluates the choice probability for each alternative because of the probability that the perceived utility of the alternative is greater than the perceived utility of the other alternatives.

The main elements of RUMs are reported in the book of Domencich and McFadden (1975); an extension to the perception of similarity between alternatives is reported in McFadden (1978) for the problem of residential location. Several theoretical aspects relative to RUMs are reported in the books of Daganzo (1979), Manski and McFadden (1981), Ben Akiva and Lerman (1985), and Cascetta (2009).

In the transport field, the demand model simulates how user behaviour is influenced by the level of services. In RUMs, the user knows the available alternatives and associates to each alternative measurable characteristic. An alternative chosen by the user derives from a sequence of choices. The sequence of choices is relative to the levels of trip (i.e., departure time interval, destination, mode, and route) and/or inside a single level (i.e., the adaptive decision at destination or during the route). For instance, let us consider a user who decides to travel for leisure purposes. It is possible to assume that, pre-trip, the user chooses an area to visit but that the same user frequently does not have a specific final destination within that area. In this case, the user has not made a unique decision before the trip; during the trip or in the destination area, the user chooses one of the alternatives or a sequence in the timing of the alternatives. Another example could be relative to a user who, before the trip, does or does not have a clear idea of the path to follow to reach his final destination; during the trip, the user chooses one of the pre-trip perceived paths.

If the user has no intermediate decision level, the choice is made when the trip starts. If the user has at least one intermediate decision level, choices are made when the trip starts and during

http://dx.doi.org/10.1016/j.tbs.2015.07.003

the trip. In this last case, assuming a steady state for the system, decisions can be

(i). unique in the intermediate levels (only one chosen pre-trip state in the intermediate levels),
(ii). not unique in the intermediate levels (more than one chosen pre-trip state in the intermediate levels).

Assuming a steady state for the system, an analyst obtains the same values of the choice probabilities in the two cases with RUMs. The same concept can also be expressed considering that, in RUMs, the choice is the result of the user's choice in the time; in the time the user moves from a state to another state; the user's decision cannot stay in two different states in the same time.

A new class of models that yields different probability values when considering the two previous cases (I and II) must be specified. It derives from quantum mechanics models in which a wave function defines the probability to stay in a particular state. The user behaviour is not similar to the particle's behaviour; quantum theory allows defining a new specification for the choice probability. This specification is defined as the sum of a probabilistic term (that does not depend on the unique or not-unique choice in the intermediate levels) and an interference term (that considers the possible not-unique choice in the intermediate levels). With this model, the analyst can evaluate the choice percentage of users for each alternative with values close to the real ones, considering that a new term is added to the probability.

Quantum Utility Models (QUMs) start from RUMs. In an analogy with RUMs, the QUMs are based on the assumptions reported for RUMs with differences related to alternatives that cannot be exclusive and to the evaluation of choice probability. In QUMs, the choice probability for each alternative is the sum of the probability that the perceived utility of the alternative is greater than the perceived utility of the other alternatives and the interference term connected to the level of interference of the specific alternative with respect to the others alternatives.

In relation to QUMs, one of the first specifications of transport for car driver behaviour was proposed by Baker (1999). Recently, Busemeyer and Bruza (2012) proposed quantum theory for cognitive and decision processes. In Wallace (2002), an analysis of quantum mechanics for decision theory is reported. A comparison between Markov processes and quantum theory is reported in Busemeyer et al. (2006). In Agrawal and Sharda (2013), quantum mechanics connected with a quantum decision is reported; quantum mechanics is described also considering some practical examples, with two appendices related to elements of quantum mechanics for application in QUMs.

In this paper, the RUMs and the QUMs are specified and compared for a choice level of the demand model in transport systems. In a demand model, a choice can have different levels in relation to the combination of mobility and trip decision (departure time, destination, mode, path and service) and to the choice of the alternative (alternative-set and alternative belonging to the alternative-set). The final choice results from the mobility and trip decision, and in each mobility and trip decision, an alternative is chosen. Commonly, several inner decision levels are considered. The advancement of this paper is relative to a specification of a new model for route (and service) choice level. This model has a specification that allows easy evaluation of the probability in the two cases (unique and not-unique decision in the intermediate levels). The QUMs could be extended to the upper levels of the transport demand model (modal split, distribution, and departure time). The model is applied in two test systems to verify applicability and to compare results with consolidated models.

In Section 2, the RUMs are reported; the RUMs are compared with the QUMs in Section 3. A specific model for route choice level and some numerical results in two test systems are reported in Section 4. In Section 5, further developments are reported.

## 2. Random utility models

In relation to the RUMs, in the first part of this section, a choice with one level of decision is considered; it is assumed that there is no intermediate decision level. In this case, there is no difference in the results between RUMs and QUMs. In the second part, it is assumed that there is one intermediate decision level. In this case, there is a difference in the results between RUMs and QUMs. The problem can be extended in the case of multi-decision levels.

### 2.1. One level of decision

In demand models with one set (mono-set) of available alternatives, named elementary set $H$, the choice probability associated to each alternative must be specified. A generic class of users, $n$, perceives a set $H$ containing alternatives $k$. In this context, the decision is assumed pre-trip, and it is not possible to change the alternative during the trip. In this case, there is no intermediate decision level. The RUMs and the QUMs yield the same result.

For each alternative $k$, the class of users n perceives a utility $U_{k,n}$. The utility is not completely known by the analyst. In relation to the assumption of the random probability distribution for $U_{k,n}$, RUMs can be specified. In this context, the choice probability, $p_n(k|H)$, for the class of users $n$ for the alternative $k$ condition to belong to the elementary set $H$, is the following:

$$p_n(k|H) = probability(U_{k,n} > U_{i,n}, \quad \forall k, i \in H, \quad k \neq i)$$

Considering a class of users $n$, if the $U_{k,n}$ values are assumed independent and identically distributed with Gumbel variates with parameter $\theta$, then the Logit model (Dial, 1971) is obtained. If the $U_{k,n}$ values are assumed Normally distributed of variance covariance matrix $\Sigma$, then the Probit model (Daganzo, 1979) is obtained. For the route choice level, these models are specified in Section 4.

In these models, it is assumed that the probability depends on the parameter of the distribution and on the expected value of the utility, $E(U_{k,n})$, assumed as follows:

$$E(U_{k,n}) = V_{k,n} = \sum_{j=1}^{m} \beta_{j,k,n} \cdot V_{j,k,n} \quad \forall\, k \in H$$

The expected value is calculated as a function of $m$ system-measurable characteristics $y_{j,k,n}$, associated to alternative $k$ and class of users $n$, through some $\beta_{j,k,n}$ coefficients (that must be calibrated). The parameters of the probability distribution very often are included inside the expected value of the utility.

### 2.2. Two levels of decision

In a real context, decisions very often are mixed pre-trip and en route; users choose one or more alternative-sets before the trip and choose the alternative-set and the alternative with a mixed pre-trip and en route decision. In two or more decision levels, users have more than one decision for each alternative $k$.

In this paper, the case of multi-decision levels is considered with one alternative-set (mono-set). The case of more than one alternative-set (multi-set) can be considered. For simplicity, it is briefly reported in Appendix A.

When only one alternative-set is considered, a multi-decision level can be represented with a decision tree and modelled with a Markov process. The transition probability to level (or event) $j$ from level (or event) $i$ is indicated with $p(j|i)$. The transition probability can be simulated with a RUM.