A Bayesian perspective on magnitude estimation

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Our representation of the physical world requires judgments of magnitudes, such as loudness, distance, or time. Interestingly, magnitude estimates are often not veridical but subject to characteristic biases. These biases are strikingly similar across different sensory modalities, suggesting common processing mechanisms that are shared by different sensory systems. However, the search for universal neurobiological principles of magnitude judgments requires guidance by formal theories. Here, we discuss a unifying Bayesian framework for understanding biases in magnitude estimation. This Bayesian perspective enables a re-interpretation of a range of established psychophysical findings, reconciles seemingly incompatible classical views on magnitude estimation, and can guide future investigations of magnitude estimation and its neurobiological mechanisms in health and in psychiatric diseases, such as schizophrenia.

Theories of magnitude estimation

Our ability to judge duration, distance, or size is crucial for a mental representation of, and interaction with, the physical world, such as building a cognitive map, performing accurate movements, playing an instrument, or doing sports [1,2]. It has long been known that humans show strikingly similar behavioral signatures (and biases) in magnitude estimation across different sensory modalities, such as proprioception, vision, or audition [3–9]. Along with imaging studies, the universal expression of these behavioral effects has supported the idea of a generalized magnitude estimation system [10–14]. However, at the same time, each physical quantity might also have a specialized representation that is related to the sensory organs with which it is typically associated and the computational problems in whose treatment it has a role [15]. Therefore, previous work has called for computational models as a way to disentangle common and distinct processes in magnitude representation and estimation [16].

So far, however, attempts to model magnitude estimation have often led to modality-specific or effect-specific explanations [17]. By contrast, recently proposed Bayesian accounts of magnitude estimation have the potential to provide a more general explanation that covers a wide set of behavioral characteristics and transcends any specific modality [18–20]. This Bayesian framework suggests that behavioral phenomena of magnitude estimation, such as characteristic biases observed across sensory domains, are the result of integrating noisy sensory information with prior experience. From this perspective, estimation errors are neither due to limitations of the sensory channels nor result from erroneous cortical representations. Instead, on average, they optimize behavioral outcomes by accounting for noise and are the natural consequence of general principles underlying perceptual inference (i.e., the deployment of a predictive model that takes the learned statistics of the environment into account) [21]. This perspective derives from long-standing theories of perception in general and provides a formal foundation to examine aberrations of magnitude estimation in psychiatric diseases, such as schizophrenia [22].

In this review, we discuss how a Bayesian framework can: (i) provide a unifying perspective that explains a variety of behavioral features of magnitude estimation; (ii) shed new light on classical psychophysical laws by reconciling the work of Weber-Fechner and Stevens and

Glossary

Discrimination task: requires binary decisions about the difference between two consecutively or simultaneously presented stimuli (e.g., whether tone A is louder than tone B).

Generative model: specifies a joint probability distribution of hidden states and/or parameters and the observed data; this requires specification of likelihood and prior.

Kalman filter: a statistical technique, which infers the current (hidden) state of a state space model based on the previous observations. It can be used to model an online Bayesian estimation process that is updated on a trial-by-trial basis.

Matching task: requires that the magnitude of a new stimulus is actively adjusted to a previously experienced one. Matching tasks can be used within the same stimulus dimension ('within-modality matching'), such as reproducing a walked distance, or across different modalities ('cross-modality matching'), such as matching a number to the brightness of a light bulb.

Stevens' power law: proposes a power law relation between physical magnitudes and the representation by sensory systems. The power law exponent is characteristic for the respective sensory modality.

Weber-Fechner law: proposes a logarithmic relation between physical magnitudes and the representation by sensory systems.

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providing a re-interpretation of their laws; and finally (iii) guide the exploration of the neurobiological underpinnings of magnitude estimation in health and disease.

A Bayesian framework for magnitude estimation

Regardless of whether we examine the estimation of time, distances, length, or loudness, certain behavioral phenomena reoccur across studies (Figure 1A) [23]. The most common ones are depicted in Figure 1B: (i) A tendency of subjective estimates to be biased towards the center of the distribution (regression effect); (ii) an increase of this bias for larger sample ranges (range effect); (iii) a linear increase in standard deviation of estimates with mean magnitude (scalar variability); and (iv) correlations between subsequent magnitude judgments (sequential or order effects) (see Box 1 for a detailed description). Although scalar variability seems to be the consequence of a general logarithmic representation of magnitudes according to the Weber-Fechner law [24] (see Box 2 and Glossary), the remaining effects have often only been explained by modality-specific theories [17].

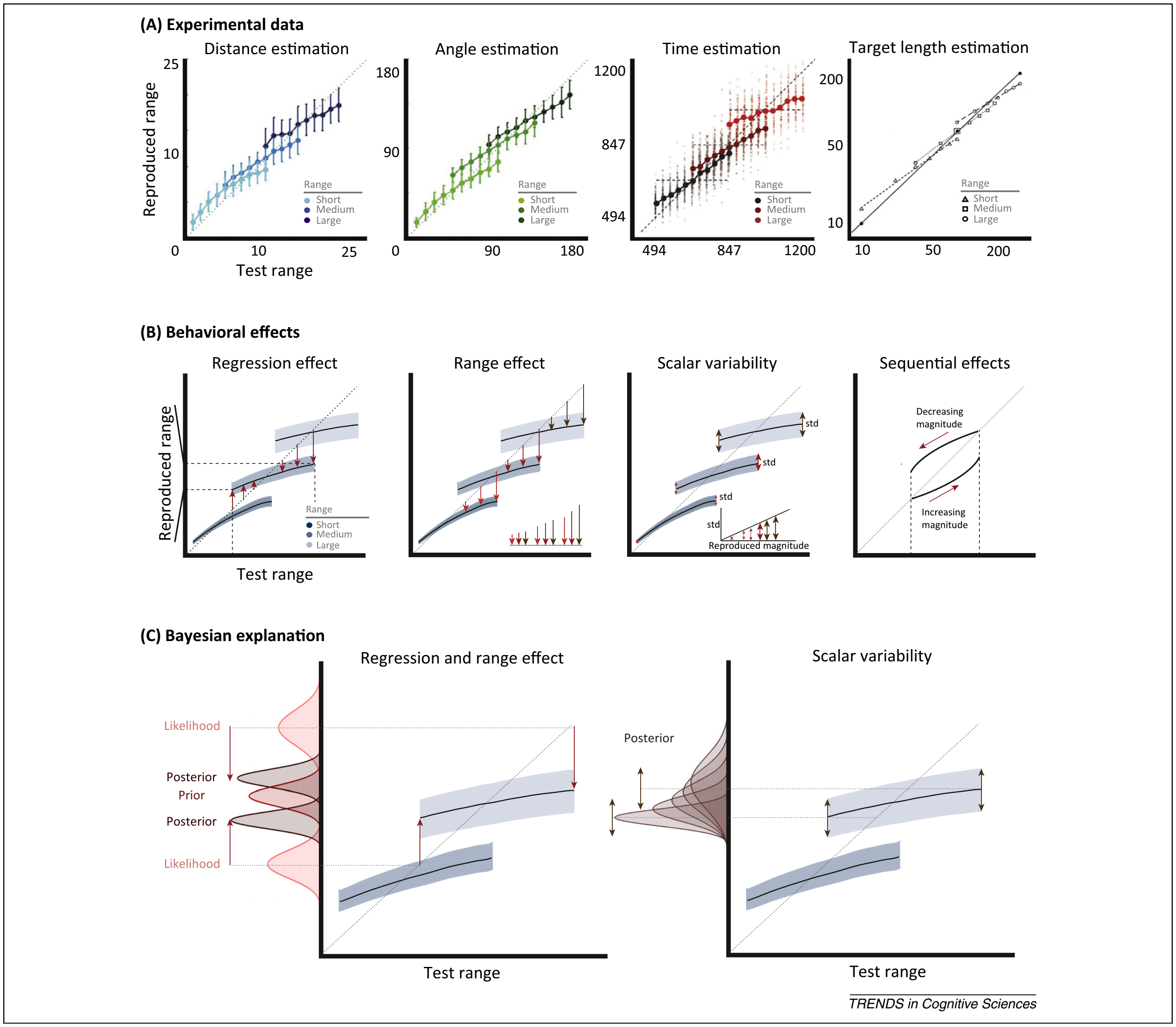

Figure 1. Overview of the behavioral signatures in magnitude estimation and their Bayesian explanation. **(A)** Similar behavioral characteristics observed in data from four different magnitude estimation experiments over more than 100 years on distance estimation, turning angle estimation, time estimation, and target length of a guided movement [18,19,103,104]. Each study used three test ranges (short, medium, and large range). Note that length estimation is plotted on logarithmic scales; therefore, there is no characteristic curvature in the reproduction data. **(B)** Detailed depiction of the observed behavioral characteristics in magnitude estimation from (A). The regression effect refers to a characteristic bias towards the center of each test range, leading to a smaller reproduced range compared to the physical test range. The range effect refers to an increase of the regression effect for larger sample ranges. Scalar variability refers to a linear increase in standard deviation with the mean of the reproduced magnitude. Sequential effects refer to a bias in magnitude estimates towards the recent history of stimuli experienced (see also Box 1). **(C)** A Bayesian framework can explain the characteristics effects shown in (B). A prior around the center of the test distribution would bias posterior estimates towards the center of the respective test range, causing the range and regression effect. Scalar variability predicting an increase in standard deviation with the mean of the likelihood would cause the bias to be stronger for larger magnitudes (larger sample ranges). On all plots: the tested sampled magnitudes are on the X-axis and estimated reproduced magnitudes are on the Y-axis. Diagonal lines reflect the location of a nonbiased, veridical magnitude estimate. Adapted from [18,19,103] (A).