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Accuracy and precision of fractal dimension measured on model surfaces

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ABSTRACT

Objectives. To develop a method, which is precise, accurate, and insensitive to the angle of inclination for determining the fractal dimensional increment (D^*) of a surface.
Methods. Brownian interpolation was used to generate simulated ceramic fracture surfaces having known D^* values of 0.1, 0.2, 0.3, and 0.4 with 10 surfaces at each D^* value. Each surface was inclined at four angles (0°, 3°, 5°, and 7°) from horizontal. The 160 (40×4) surfaces were analyzed by a variety of methods including Minkowski Cover (MC), Root Mean Square Roughness vs. Area (RMS), Kolmogorov Box (KB), Hurst Exponent (HE), Slit Island Box (SIB), and Slit Island Richardson (SIR). The coefficient of variation (CV) and mean error were used to identify the methods with best precision (lowest CV) and accuracy (lowest mean error), respectively, and three-way ANOVA followed by Turkey's HSD ($\alpha = 0.05$) was used to identify significant effects.
Results. CV was significantly affected by fractal dimension ($p = 0.002$) and method ($p < 0.001$) but not by angle of inclination ($p = 0.765$). The CV value for MC was lower than those for other methods ($p \leq 0.05$). Mean error was significantly affected by three-way interaction between fractal dimension, method, and angle of inclination ($p < 0.001$). The mean error for KB was higher than those for other methods ($p \leq 0.05$) for inclined surfaces.
Significance. MC was determined to have the best combination of precision, accuracy, and lack of sensitivity to angle of inclination for Brownian interpolation surfaces having D^* values in the range commonly reported for ceramic fracture surfaces.

1. Introduction

In order to quantitatively describe the topography of natural surfaces Mandelbrot reintroduced the concept of fractal geometry [1], a non-Euclidean geometry that allows for non-integer dimensions in which fractal objects may exhibit both self-similarity (meaning that multiple features on the surface of an object appear the same in each of the X, Y and Z axes) and scale invariance (meaning that objects appear the same at all scales of magnification) [2,3]. Fractal objects are characterized by their fractal dimension, D, and by its fractal dimensional increment, D^* [1]. For any given fracture surface, D lies somewhere between a value of 2.0 and 3.0. D^* is equal to the non-integer portion of D and lies somewhere between 0 and 1 [4] (Fig. 1). D^* represents the degree of tortuosity of the surface outside of the plane [3] and serves as a scaling factor for both the fracture energy and the surface area of the created fracture surface [5].

The ability of fractal geometry to simply describe otherwise complex surfaces has led to its application in many fields of chemistry, physics, engineering, computer science

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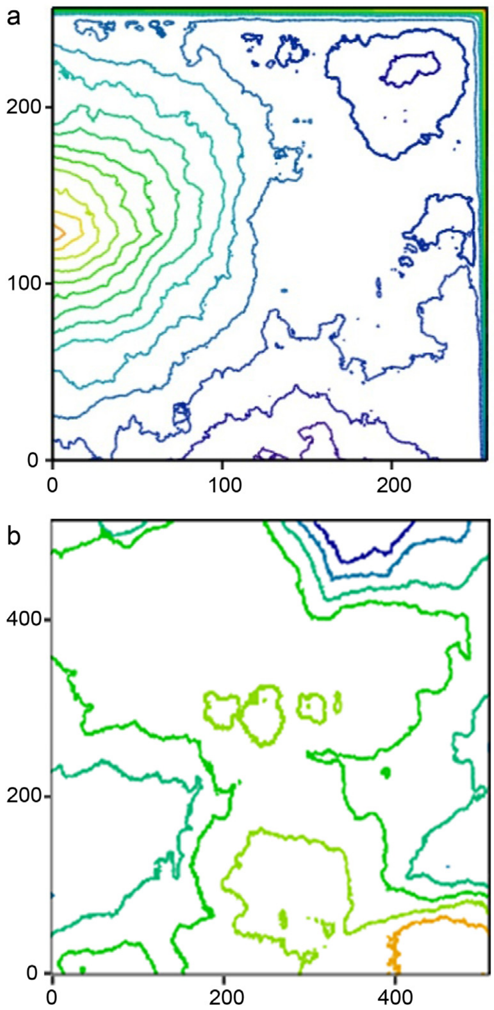

Fig. 1 – (a) Surface generated via Brownian interpolation with FRACTALS program. (b) Fracture surface of Y-TZP (ZirCAD, Ivoclar Vivadent).

and materials science [3,6]. The brittle fracture of materials has been shown to be a fractal process by numerous experiments [2,5,7–12]. The fracture process for brittle materials begins with the separation of primary bonds at the atomic level [5]. Due to the scale invariant nature of fractal geometry, the results of processes can be deduced on the atomic scale by examining specimens on the microscopic scale [2]. Having the ability to determine fracture surface geometry is necessary to gain increased understanding of the toughening mechanisms, fatigue resistance, and role of microstructure in the fracture processes of brittle solids [13]. In addition to describing the tortuosity of a fracture surface [14–17], the fractal dimension of the fracture surface can be used to determine fracture toughness [2,8,14,18–20], impact energy [21], characteristic length [2,12,22,23], and flaw/fracture mirror size ratio [20,24,25]. An advantage of fractal analysis is that it offers a method to analyze in vivo failures of dental biomaterials to distinguish whether failure of the prosthesis was due to mechanical overload or the fabrication process [6]. Traditional failure analysis of dental restorations often proves difficult because further damage is created during the clinical retrieval process, and important portions of the fracture surface (i.e. critical flaw/failure origin) are often lost [26]. This may be caused by applied iatrogenic pressure during retrieval of the broken prosthesis or by continued loading following the fracture event. The fractal dimensional increments, D^*, of several dental ceramics have previously been investigated [27].

Fractal objects are either self-similar or self-affine. To differentiate, self-similar objects show the same dimensions in the Z direction scale as those in the X and Y; whereas, for self-affine objects the fractal dimension of the vertical direction is different from the lateral directions [24]. This results in a different D^* value when measurements are made outside of the horizontal plane [22]. It is commonly accepted that fracture surfaces are self-affine [9,22,28].

Many different methods are used for measuring the fractal dimension of a surface. Some can measure an original fracture surface [22,29–37], while others take a zero-set approach (slicing a cross-section of the original surface) [1,38–40]. Zero-set methods have been the most commonly used methods for dental materials [23,24,27,41]. The majority of researchers have used a zero-set method called slit-island analysis [3,4,23,42–46] to acquire a horizontal section of a fracture surface followed by the Richardson method [38] to determine the fractal dimension. However, two major obstacles arise when using Slit-Island Richardson (SIR) method [9,22,42,46–52]. First, the SIR method is an extremely tedious and labor intensive technique when human judgment is used to trace a micrograph of the cross-section, and no automated method has been validated [23]. Also, since the D^* of a self-affine surface will only match the D^* of its zero-set when the slicing angle is 0°, a fracture surface can only be analyzed accurately when measured in a plane exactly parallel to the surface, which is difficult to achieve manually [24].

Most methods for measuring fractal dimension have been used only on fracture surfaces of physical specimens [2,3,5,13,19,23,24,27,42,53–55]. Here an obstacle arises because the true D^* values of the analyzed surfaces are unknown since there is no standard reference material for the D^* of fracture. Therefore, the present study sought to synthetically generate fractal surfaces with known D^* values to determine which methods of fractal analysis are the most accurate, precise, and robust. Several other studies have used similar computer-generated surfaces to measure fractal dimension in a systematic way [56–59]. These studies used a limited number of fractal analysis methods to determine the most accurate methods, and none of them considered angle of inclination. While these studies have concentrated on finding the measurement methods with the greatest accuracy [56,57,59], one focus of this study is to find a method with the highest precision. Precision is advantageous over accuracy because a correction factor may be applied to compensate for lack of accuracy and hence to achieve an unbiased a result. The objective of this study is to develop a method for determining the fractal dimensional increment (D^*) of a surface which is precise, accurate and insensitive to angle of inclination. Such a method would be a valuable tool for failure analysis, production, and development of new dental materials [60].