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Non-inverse-square force–distance law for long thin magnets—Revisited

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ABSTRACT

Objective. It had previously been shown that the inverse-square law does not apply to the force–distance relationship in the case of a long, thin magnet with one end in close proximity to its image in a permeable plane when simple point-like poles are assumed. Treating the system instead as having a 'polar disc', arising from an assumed bundle of dipoles, led to a double integral that could only be evaluated numerically, and a relationship that still did not match observed behavior. Using an elaborate 'stretched' exponential polynomial to represent the position of an 'elastic' polar disc resulted in a fair representation of the physical response, but this was essentially merely the fitting of an arbitrary function. The present purpose was therefore to find an explicit formula for the force–distance relationship in the polar-disc problem and assess its fit to the previously obtained experimental data.

Methods. Starting from Coulomb's law a corrected integral formula for the force–distance relationship was derived. The integral in this formula was evaluated explicitly using rescaling, changes of order of integration, reduction by symmetry, and change of variables. The resulting formula was then fitted to data that had been obtained for the force exerted by eighty-five rod-shaped magnets (Alnico V, 3 mm diameter, 170 mm long) perpendicular to a large steel plate, as a function of distance, at small separations (< 5 mm). Subsequently, the fit of alternative functions was explored.

Results. An explicit formula in terms of elliptic integrals was obtained for the polar-disc problem. Despite the greater fidelity, this too was found not to fit the observed physical behavior. Given that failure, nevertheless a simple formula that conforms closely and parsimoniously to the actual magnet data was found. A key feature remains the marked departure from inverse-square behavior.

Significance. The failure of the explicit formula to fit the data indicates an inadequate model of the physical system. Nonetheless it constitutes a useful tool for quantifying the force–distance relationship on the premise of polar discs. Given these insights, it may now be possible to address the original motivating problem of the behavior of real dental magnets.

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1. Introduction

Miniature magnets are used in dentistry for functions of denture retention and orthodontic treatment, and there is some interest in understanding the relationship between the force exerted on a keeper and its separation from a magnet. There is considerable disagreement in the dental literature as to the nature of that relationship, and experimental determinations are somewhat imprecise and contradictory [1]. For example, it has been said to be approximately inverse-square [2], inverse square-root [3], inverse square then inverse cube [4], inversely proportional [5], and inverse power law with a power between one and two [6]. It is the purpose of this work to resolve this confusion.

As a simplification, the behavior of long, thin cylindrical (170 mm long and 3 mm diameter) magnets against a magnetizable stainless steel disc was studied in [1], in which work an attempt was made to allow for the fact that a point-like pole, while often a convenient modeling device, could not be assumed in practice. Key conclusions were (i) that the inverse-square law for a point-like pole was found not to apply to the experimental data in any region tested, (ii) that a polar disc model was found to provide a very good fit to the experimental data over the whole range tested, but only when an offset (≈1 mm) of the pole position from the magnet face was assumed, and (iii) that this offset experienced a stretched-exponential pole-position relaxation (over typically ≈0.4 to 1.5 mm). Modeling was indirect through a polynomial function approximation to a numerical evaluation of the (then) apparently intractable integral, which was modified to accommodate the offset and movable 'polar disc'.

While the key result of non-inverse-square law behavior may have clarified the source of the experimental problems that had previously been experienced, the lack of a suitable explicit expression for the relevant integral was unsatisfactory. Accordingly, this was reconsidered, with a view to identifying a cleaner approximating function for more practical application, bearing in mind that a full theoretical model—while of fundamental value—would not necessarily lend itself to the context of interest.

2. The explicit formula

On the assumption that Coulomb's law holds, and that in effect a sufficiently long real cylindrical magnet can be treated as a disc-shaped continuum of poles when one end is close enough to a permeable plane, as previously taken as the basis in [1], the derivation described in Appendix A yields the following for the force, F:

$$F = \frac{\mu_0 QQ'}{4\pi(2d)^2}J, \tag{1}$$

where μ_0 is the magnetic permeability of free space ($4\pi 10^{-7}\,\mathrm{Hm^{-1}}$), Q the strength of the magnet, Q' the strength of its image, d the separation *in vacuo* of the magnet and its image from a reference point midway between them (*i.e.* on the permeable plane), and J a dimensionless variable determined by the geometry of the configuration. If the end of the magnet is representable by a point-like pole, then one would have $J = 1$. For the case of a circular disc of radius R,

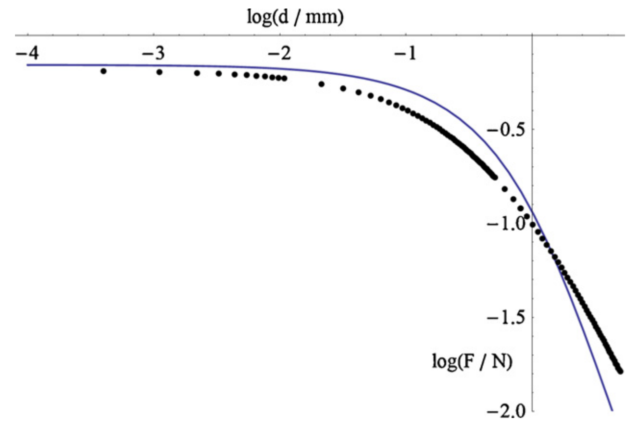

Fig. 1 – Example of the failure of the fit of formulae (1)–(4) (solid curve) to magnet data (filled circles).

$$\begin{aligned} J \;=\; & \frac{4}{\rho^2} - \frac{2(2\rho^2+1)}{\pi\rho^4\sqrt{4\rho^2+1}}K(k) + \frac{2\sqrt{4\rho^2+1}}{\pi\rho^4}E(k) \\ & - \frac{4(1-\nu_+)}{\pi\rho^2\sqrt{4\rho^2+1}}\Pi(\nu_+, k) \\ & - \frac{4(1-\nu_-)}{\pi\rho^2\sqrt{4\rho^2+1}}\Pi(\nu_-, k), \end{aligned} \tag{2}$$

where

$$\rho = \frac{R}{2d}, \tag{3}$$

$$k = \frac{4\rho^2}{4\rho^2+1}, \quad \nu_\pm = 2\rho(\pm\sqrt{\rho^2+1}-\rho), \tag{4}$$

and K, E and Π are the complete elliptic integrals of the first, second and third kinds, defined by (A.8), (A.9) and (A.10), respectively. Although the complete elliptic integrals do not fall into the category of elementary functions, they are standard special functions of mathematical physics and readily computable. See, for instance, the solid curve in Fig. 1.

It transpires from the analysis leading to (1)–(4) that the integrand in equation (10) in ref. [1] is faulty, missing a factor xy. As a consequence, the corresponding factor J is not dimensionless. Moreover, as the separation of the disc and its image becomes greater and greater, or alternatively as the radius of the disc becomes smaller and smaller, one would expect that the behavior of the magnet and its image more and more closely resemble that of a dipole. In other words, one would expect that $J \to 1$ as $\rho \to \infty$. This is true of (2) (for details, see Appendix A). However, the counterpart in [1] is such that $J \sim 4R^{-2}$ as $\rho \to \infty$. Similarly, as the separation diminishes, one would expect that J would decrease proportionally to ρ^{-2}, so that the separation force F would attain a maximal (finite) value upon contact. The expression (2) is such that $J \sim 2\rho^{-2}$ as $\rho \to \infty$ (again, see Appendix A for details). On the other hand, the corresponding expression in [1] has the property that $J \sim 4R^{-2}\rho^{-2}\ln\rho$ as $\rho \to \infty$, resulting in an infinite

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