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Simple solutions of multilayered discs subjected to biaxial moment loading

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ABSTRACT

Objectives. The purpose of this study was to derive a simple closed-form solution for the stress distribution through the thickness of multilayered discs subjected to biaxial moment loading, such that it can be used readily to evaluate the biaxial strength of multilayered dental ceramics using biaxial flexure tests.

Methods. A simple analytical model was developed to derive the stress distribution through the thickness of multilayered discs subjected to biaxial moment loading. The accuracy of the solution was verified by comparing with previous rigorous analytical solutions and finite element results. The results obtained from Roark's formulas for bilayered discs were also included for comparison.

Results. Examples of porcelain/zirconia bilayered discs subjected to ring-on-ring and piston-on-ring loadings were used for comparison among different analyses. Despite the simplicity in deriving the present solution, it is sufficiently accurate in comparing with previous rigorous solutions and finite element results. Also, if the biaxial stresses on the top and the bottom surfaces of the disc can be measured during testing, the biaxial stress/strain through the entire thickness of the multilayered disc can be determined using equations derived in the present model.

Significance. International Organization for Standardization has selected piston-on-three-ball tests to establish ISO 6872 for dentistry-ceramic materials. However, monolayered specimens are considered in tests for this standard. While dental materials are usually fabricated into layered structures, modification of the current standard is essential. Our simple closed-form solution serves as a basis for extending the current standard to multilayered systems.

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1. Introduction

It has been noticed that when bend bars are fabricated from dental ceramics for uniaxial strength measurements, defects are commonly formed due to the processing constraints presented by bend bars [1]. These defects are not found in clinical dental crowns, and they are also absent in disc specimens that are used for biaxial strength measurements. Also, in most prosthetic applications, dental crowns function in a state of biaxial stress. As a result, biaxial moment loadings, which can be achieved using biaxial flexure tests such as ring-on-ring, piston-on-ring (or piston-on-three-ball), and ball-on-ring tests are becoming increasingly popular for measuring the biaxial strength of dental ceramics [2–14]. In this case, a thin disc is supported by a ring (or three balls) near its periphery and loaded through a smaller coaxial ring, a piston, or a ball in its central region. The disc is subjected to a biaxial moment in its central region and the stresses are biaxial in this region.

International Organization for Standardization has selected piston-on-three-ball tests to establish ISO 6872 for dentistry-ceramic materials [15]. This selection is justified because the piston-on-three-ball fixture does not require absolute planarity of the two surfaces of the disc. Also, it has been proved by Kirstein and Woolley [16] that the biaxial stresses in the central region of the disc are the same regardless the disc is supported by a ring or three balls. Hence, the stress solutions which were originally derived for piston-on-ring loading could also be used for piston-on-three-ball loading. However, the application of ISO 6872 has been limited to the case of monolayered discs. While all-ceramic dental crowns are usually fabricated into layered structures with esthetic but relatively weak veneer porcelains on stiff and strong ceramic supporting-cores, modification of the current standard to extend its applicability to multilayered discs is essential.

Roark's formulas [17] for bimetallic circular plates subjected to biaxial moment loading have been used to evaluate the biaxial strength of bilayered discs. However, only the equations for biaxial stresses on the top and the bottom surfaces of the disc were given in Roark's formulas. Depending upon the elastic constants and thicknesses of the constituent layers, the maximum tension can occur at the interface rather than the surface in a multilayered disc. In this case, the maximum tension cannot be obtained from Roark's formulas. Recently, Hsueh et al. [18–22] derived closed-form solutions for stresses in multilayered discs subjected to biaxial flexure tests and the accuracy of solutions was verified by finite element results. Although they are closed-form solutions, the expressions are lengthy which may deter the engineers, who are less adept in mechanics and mathematics, from using them. Hence, the purpose of the present study is to derive a simple closed-form solution for the stress distribution through the thickness of multilayered discs subjected to biaxial moment loading, such that it can be used readily by engineers to evaluate the biaxial strength of multilayered dental ceramics using biaxial flexure tests. First, Roark's formulas for bilayered discs and Hsueh et al.'s previous rigorous closed-form solutions for multilayered discs are summarized to provide a general background. Then, the derivation of the present simple solution is shown. Finally, specific results are calculated for zirconia monolayered and porcelain/zirconia bilayered discs subjected to ring-on-ring and piston-on-ring loadings. Comparison among the three sets of formulas and the finite element results is made to examine the accuracy of the present solution.

2. Materials and methods

2.1. *Multilayered discs subjected to biaxial moment loading*

A diametrical section through the axis of symmetry of a thin multilayered disc is shown schematically in Fig. 1. The disc consists of n layers with individual thickness, t_i, where the subscript, i, denotes the layer number with layer 1 being at the bottom. The cylindrical coordinates, r, θ, and z, are used. The lower surface of layer 1 is located at $z=0$, the interface between layers i and $i+1$ is located at $z=h_i$, and the upper surface of layer n is located at $z=h_n$. With these definitions, h_n is the thickness of the disc, and the relation between h_i and t_i is described by

$$h_i = \sum_{j=1}^{i} t_j \quad (i = 1 \text{ to } n). \tag{1}$$

The disc is subjected to biaxial moment loading with $z=0$ and $z=h_n$ being the support and the load surfaces, respectively, and the interfaces between layers are assumed to remain bonded during loading.

For biaxial moment loading, ring-on-ring, piston-on-ring (or piston-on-three-ball), and ball-on-ring biaxial flexure tests have been conventionally used. Derivation of the biaxial stress distribution in the disc during loading is contingent upon the determination of the following two relations: (1) the biaxial stress–biaxial moment relation and (2) the moment–load relation. While the first relation is valid for all biaxial moment loadings, the second relation depends on the individual loading configuration. Because ring-on-ring [23–25] and piston-on-ring [16,24–26] loadings on monolayered discs have been adopted, respectively, by ASTM 1499

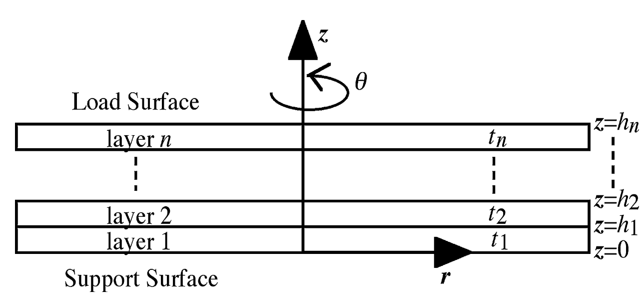

Fig. 1 – A schematic drawing of the diametrical section through the axis of symmetry of a multilayered disc showing the coordinate system used in analyses and the load and the support surfaces during biaxial moment loading.