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# Analysis of ball mill grinding operation using mill power specific kinetic parameters

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A B S T R A C T

With a view to developing a sound basis for the design and scale-up of ball mills, a large amount of data available in the literature were analyzed for variation of the two key mill performance parameters: power specific values of the 'absolute breakage rate of the coarsest size fraction', $S^*$, and 'absolute rate of production of fines', $F^*$, with some of the important operating and design variables such as the mill speed, ball load, particle load, ball diameter and mill diameter. In general, values of both the mill performance parameters were found to vary significantly with the mill operating conditions. The nature and relative magnitude of variation for the two parameters also differed significantly. Moreover, the effect of any particular variable on the $S^*$ and $F^*$ values was found to be significantly different for different sets of operating conditions. It has been emphasized that, as the purpose of grinding is to produce fine particles, the mill design and scale-up work should be based mainly on the $F^*$ parameters. Moreover, it is not correct to regard the $S^*$ values to be independent of the mill design and operating variables as a general rule, especially for a fine analysis of the performance of the grinding systems.

## 1. Introduction

The world over, every year several billion tons of metallic ores, minerals, cement and various other solids used in the ceramic and chemical industries are subjected to size reduction in ball mills. The specific energy consumption value for grinding of these materials typically ranges from 5 to 50 kW h/ton. Thus, a significantly large amount of electrical energy is consumed in the ball mill grinding operation. It is, therefore, important to establish the optimum values of various mill operating parameters, such as the mill speed, ball load, ball diameter and particle load, from the energy consumption point of view.

Another important task associated with the ball mill grinding operation is to establish a sound basis for carrying out scale-up of ball mills based on laboratory or pilot scale test work. This requires studies related to the influence of mill diameter on production rate of particles of a desired size distribution.

During the last four decades, considerable amount of work has been done pertaining to the above mentioned two tasks using a phenomenological grinding kinetics mathematical model derived from population balance considerations. This model is based on two sets of parameters: specific breakage rate and breakage distribution parameters for various size fractions of particles [1–4]. The equation used for describing batch grinding kinetics is

$$\frac{dM_i(t)}{dt} = -S_i M_i(t) + \sum_{j=1}^{i-1} b_{i,j} S_j M_j(t) \tag{1}$$

where $M_i(t)$ is the mass fraction of the particulate solids in the sieve size interval $i$ (bounded by size of the aperture of upper sieve $x_{i-1}$ and size of the aperture of lower sieve $x_i$), $t$ is grinding time, $S_i$ is specific breakage rate for particles of size class $i$ (fractional rate at which material breaks out of size interval $i$), and $b_{i,j}$ is the weight fraction of the material breaking out of sieve size interval $j$ that reports to sieve size interval $i$.

Herbst and Fuerstenau [5], Kim [6], Malghan [7], Malghan and Fuerstenau [8], Siddique [9], and Fuerstenau [10] have analyzed the variation of grinding rate of some selected sieve size fractions (such as a 10/14 mesh size fraction) of quartz, dolomite and limestone with various operating variables in batch ball mills of different diameters. They concluded that the absolute grinding rate (the product of specific grinding rate and weight of the particulate contents of the mill) per unit net power input to the mill does not vary with the mill operating conditions such as the mill speed, ball load, particle load and mill diameter. It was mentioned that the breakage distribution parameters were also to a good first approximation independent of these operating variables within the normal operating range. Their proposition can be mathematically expressed as [11,12].

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**Nomenclature**

| | |
|---|---|
| $b_{i,j}$ | weight fraction of the material breaking out of sieve size interval $j$ that reports to sieve size interval $i$. |
| $B_{i,j}$ | fraction of broken product that is finer than lower size limit of the size interval $i$ when particles of size interval $j$ undergo breakage |
| $E^*$ | energy input to the mill per unit mass of the particulate charge |
| $F^\circ$ | rate of production of fines relative to the value for the first time interval |
| $F^*$ | absolute rate of production of fine material per unit power input |
| $F^{**}$ | power specific absolute rate of production of fine material relative to the corresponding value for some chosen value of the operating condition under consideration |
| $F_i$ | mass fraction of particulate solids finer than sieve $i$ |
| $H$ | total mass of the particulate charge in the mill |
| $J$ | fraction of mill volume filled by static ball charge |
| $M_i$ | mass fraction of the particulate solids in the sieve size interval $i$ |
| $N$ | fraction of critical speed of mill |
| $P$ | net power drawn by mill |
| $S^\circ$ | grinding rate relative to the grinding rate for the first time interval |
| $S^*$ | absolute rate of breakage per unit power input |
| $S^{**}$ | power specific absolute grinding rate relative to the power specific absolute grinding rate for some chosen value of the operating condition under consideration |
| $S_i$ | specific breakage rate for particles of size class $i$ |
| $t$ | grinding time |
| $U$ | fraction of void volume of the static ball charge occupied by the particulate solids |

$$S_i = S_i^*[P/H] \quad (2)$$

where the proportionality constants $S_i^*$ are independent of mill design and operating variables, $P$ is net mill power input and $H$ is the weight of particulate contents of the mill. $S_i^*$ can also be defined as 'absolute breakage rate per unit power input' ($S_i^* = S_iH/P$). For a batch mill drawing constant power, the specific energy input to the mill $E^*$ is given by

$$E^* = Pt/H \quad (3)$$

Incorporation of Eqs. (2) and (3) in Eq. (1) gives

$$\frac{dM_i(E^*)}{dE^*} = -S_i^*M_i(E^*) + \sum_{j=1}^{i-1} b_{i,j}S_j^*M_j(E^*) \quad (4)$$

This 'energy-size reduction relationship' predicts that for a given material and feed size distribution a necessary condition for identical product size distribution in different batch mills is identical specific energy inputs into each mill – independent of mill dimensions and mill operating variables in the normal operating range [11,12].

Besides the grinding rate of the coarser size fractions, the rate of production of the desired size fine product is also an equally important parameter for characterization of the mill performance. Let the weight fraction of material finer than size $x_i$ at time $t$ be denoted by $F_i(t)$. For the rate of production of material finer than size $x_i$ we can write

$$\frac{dF_i(t)}{dt} = \sum_{j=1}^{i-1} B_{i,j}S_jM_j(t) \quad (5)$$

where $B_{i,j}$ denotes fraction of broken product that is finer than lower size limit of the size interval $i$ when particles of size interval $j$ undergo breakage. It is well known that in the batch grinding operation, when a single size feed such as a 10/14 mesh size fraction is ground, the initial rate of production of material finer than a given size remains constant for a short but significant duration of time, depending on the fineness of the chosen size. This phenomenon is known as 'zero order production of fines' [13,14]. Let us now define a new parameter 'power specific absolute rate of production of fines', $F_i^*$, as

$$F_i^* = (dF_i(t)/dt)(H/P) \quad (6)$$

Combining Eqs. (2), (5), and (6) we have

$$F_i^* = \sum_{j=1}^{i-1} B_{i,j}S_j^*M_j(t) \quad (7)$$

From Eq. (7) it follows that if both $S_j^*$ and $B_{i,j}$ parameters were independent of the mill operating conditions, then for a given size distribution of the particulate charge of the mill at time $t$, the value of the parameter $F_i^*$ should also be independent of the mill operating conditions. In case of a single size feed charge also the same should be true for the time domain corresponding to zero-order production of fines. However, analysis of some of the published data and our own data showed that this is not true. Significant variations were observed in $F_i^*$ with the mill operating conditions.

As the main purpose of grinding operation is to produce fine particles of desired size and size distribution, a study of variations in the $F_i^*$ parameters with the mill operating conditions should be of greater concern and value for carrying out mill design and scale-up. Therefore, it was decided to carry out a detailed analysis of the available experimental data with regard to the variation of the two sets of mill power specific parameters, $S_i^*$ and $F_i^*$.

## 2. Approach to analysis of experimental data

The technique commonly used for determination of the $S$ value for the top size interval is based on an assumption that the grinding rate is independent of grinding time [1–10,15–18]. Thus, the disappearance kinetics for the top size fraction in the particulate charge are first order, as expressed below

$$dM_1/dt = -S_1M_1(t) \quad (8)$$

From this equation we obtain:

$$\ln M_1(t) = \ln M_1(0) - S_1t \quad (9)$$

Generally, the product obtained from the first test is ground sequentially several times and, in accordance with Eq. (9), the $S$ value is obtained by fitting a straight line [1–10,15–18]. It has been claimed based on such graphical representation of experimental data that very frequently the first order hypothesis is an excellent approximation to the truth [4]. But, this is not confirmed by calculating the actual $S$ values for different intervals of grinding time using the following relationship

$$S_1 = [\ln M_1(t_1) - \ln M_1(t_2)]/(t_2 - t_1) \quad (10)$$