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Original Research Paper

Simulation and optimization of a two-stage ball mill grinding circuit of molybdenum ore

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ABSTRACT

A two-stage ball milling circuit for the grinding of molybdenum ore was investigated based upon the grinding kinetic model. To this end, batch grinding tests at the laboratory-scale were conducted to obtain the specific rate of breakage and the primary breakage distribution in wet and dry environments. The test results were then scaled-up to the conditions of a full-scale ball milling circuit for grinding molybdenum ores. A two-stage ball milling circuit algebra was employed to predict the capacity, circulation ratio, and size distributions. Comparison of the simulated results and the observed values showed that the model represents accurately the actual milling process in the plant. Simulation results indicated that the classification in the milling process was operated under non-optimal conditions, and that an increase in the mill output of more than 47% was possible simply by modifying the cut size of the classifier.

1. Introduction

Modeling and simulation of grinding mills are efficient tools to predict and optimize the grinding process. They provide process indices, e.g. product size distribution, throughput..., according to various operating parameters. The population balance method (PBM) is the most popular method to model the grinding process. In this model, the grinding machine is considered as a reactor that converts large particles into smaller particles over time. Size reduction is accomplished by the breakage forces acting upon the mass of powder inside the mill; the fineness of grinding depends on how long the material is retained in the mill. The breakage process is described by two basic functions: breakage rate function and breakage distribution function. This concept has been studied by many researchers since 1941, as comprehensively reviewed by Austin [1] and Austin et al. [2]. Based on the concept, there have been extensive works to investigate the grinding characteristics of various materials such as quartz, dolomite and limestone with various operating variables in batch ball mills, in terms of the changes in the breakage rate and the breakage distribution [3–9].

The PBM model has undergone considerable development, and different PBM models have been proposed and used for simulation and optimization of grinding circuits [10–14]. The main difference between these models lies in the methods adopted for determining the basic breakage parameters and the procedures for simulating industrial milling circuits. One ambiguous point is how to predict the performance of industrial mills using the data collected from laboratory tests. It has been often found that the breakage distribution is insensitive to milling conditions [10]. In such case, only the breakage rates need to be scaled-up. There are two main methods employed for this purpose: one is based on specific energy [10], and the other is based on an empirical expression developed from experimental observations [2]. In the former case, it is assumed that the breakage rate is proportional to the specific power input to the mill. Therefore, the breakage rates for the industrial mills are calculated by comparing only the power consumption between the industrial and the laboratory mill, without considering the other milling conditions such as ball sizes, ball loading, and mill rotational speed. In the latter case, all these milling operating parameters are considered in scaling-up the breakage rates using the empirical equations obtained from numerous experimental observations.

Meanwhile, there have been continuing efforts to understand the microscopic behavior of particle breakage upon impact via single particle breakage tests using a drop-weight or a pendulum; the objective has been to understand the fracture process under a range of specific energy and couple the results with collision process occurring in the mill for ultimate prediction of the mill performance. Narayanan and Whiten [15] first proposed the t_{10} procedure, where the single particle breakage function was obtained for various impact energy levels. Through the procedure,

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Nomenclature

a	apparent bypass fraction in Eq. (32)
A	S value at $x_i = x_o$ in Eq. (19)
b_{ij}	weight fraction that appears in size i from the breakage of a particle of size j
B_{ij}	cumulative form of b_{ij}
c	wet/dry coefficient
C	circulation ratio $= T/Q$
C_1, C_2, C_3, C_4, C_5	scale-up factors in Eqs. (21)–(26)
d	ball diameter
d_{ij}	transfer function, i.e., weight fraction of the feed of size j transferred by breakage to product size i
D	mill diameter
e_j	see Eq. (6)
f_i	weight fraction of size i in mill feed stream (see Figs. 1 and 2)
F	mass flow rate of mill feed stream (see Figs. 1 and 2)
g_i	weight fraction of size i in coarser stream of the preclassifier (see Figs. 1 and 2)
G	mass flow rate of coarser stream of the preclassifier (see Figs. 1 and 2)
g'_i	weight fraction of size i in circuit feed stream (see Figs. 1 and 2)
G'	mass flow rate of circuit feed stream (see Figs. 1 and 2)
H	mill length
i	size interval index
j	size interval index
J	ball loading
k	ball size index
K_o	overfilling factor: the ratio of output Q to the output for a filling of $U = 1$
m_k	weight fraction of the balls of size k
N_o, N_1, N_2, N_3	parameters in Eqs. (22)–(24)
p_i	weight fraction of size i in mill product stream (see Figs. 1 and 2)
P	mass flow rate of mill product stream (see Figs. 1 and 2)
q_i	weight fraction of size i in finer stream of the postclassifier (see Figs. 1 and 2)
Q	mass flow rate of finer stream of the postclassifier (see Figs. 1 and 2)
q'_i	weight fraction of size i in circuit product stream (see Figs. 1 and 2)
Q'	mass flow rate of circuit product stream (see Figs. 1 and 2)
s_1, s_2, s_3, s_4	selectivity of the classifiers (see Figs. 1 and 2)
S_i	specific rate of breakage of material of size i
SI	sharpness index of the classifier, defined by the ratio of the sizes at which $s_i = 0.25$ and 0.75
t, T	size distribution and mass flow rate of return stream of the postclassfier (see Figs. 1 and 2)
U	powder loading
V	mill volume
$w_i(t)$	weight fraction of size i at grinding time t in batch grinding
W	mill hold-up
x_i	size of particles in interval i
x_o	standard particle size in Eq. (19)
x_{50}	cut size of the classifier in Eq. (33)
x_ζ	parameter in Eq. (32)
α	parameter in Eq. (19)
β	parameter in Eq. (20)
γ	parameter in Eq. (20)
λ	parameter in Eq. (32)
Λ	parameter in Eq. (19)
μ	parameter in Eq. (19)
τ	mean residence time
φ	parameter in Eq. (20)
ϕ_c	fraction of the critical speed at which a mill is run
Φ	a function related to the residence time distribution

material-specific breakage resistance is characterized by so-called ore breakage characteristic parameters, and the result is then used to determine the product size distribution using the t_{10}–t_n relationships and the mass-size balance. This approach was adopted to describe the breakage test data by various researchers [16–24]. The modified model of the energy-t_{10} relationship recently proposed by Shi and Kojovic [25] enables to incorporate the size-dependence of breakage in the model. Recently Shi and Xie used the t_{10} procedure to simulate the batch [26] and continuous [27] grinding process in a ball mill.

In the t_{10}-based model, specific energy is a major input variable that determines the ultimate product size distribution; the mean specific energy measured by an energy meter was used in the simulation. However, in a ball mill, the particles do not receive constant energy, but a range of energy levels that induces different types of breakages, such as impact breakage and abrasion/chipping breakage. Further, it was assumed that the mean specific energy was not applied to all particles evenly in a ball mill, and was proportioned to particles of different sizes at arbitrary ratios obtained by fitting the data to the model predictions. Moreover, they seem to overlook the fact that the particles are subject to re-breakage, by which the breakage product becomes finer as the particles stay in the mill for a longer time. It is not certain that their model accurately describes the breakage phenomena occurring in ball milling.

A more sophisticated breakage modeling has been evolved using the discrete element method (DEM). DEM is a numerical method for computing the motions and collisions of particles, allowing the calculation of impact forces acting on the balls. Since the introduction of DEM in ball mill simulation [28], there have been continuing efforts to integrate DEM collision energy spectra with PBM for the prediction of product size distribution in ball milling [29–32]. A typical approach taken is to relate the DEM output with the experimental data, and deduce the breakage rate and breakage distribution that are then coupled with the energy-based population balance model. However, this technique has not reached a level of refinement for being used in all grinding applications, because of the complex breakage mechanisms involved in the grinding devices that are operated under various grinding circuit configurations.

In this study, a two-stage ball mill grinding circuit currently run at the molybdenum mine in Jecheon, Korea, is investigated. The concept of this work is based on the kinetic grinding model, with the methodology proposed by Austin, Klimpel and Luckie [2]. This method is not based on specific energy, but is capable of predicting the mill performance of various grinding circuits. Unlike what many researchers had depicted, the breakage parameters are determined experimentally, and not by back-calculation. In the t_{10} procedure, the grinding process is considered as an event process, and only the resulting breakage product at various energy inputs is of interest. In Austin's approach, the grinding process is considered as a rate process, and the breakage rate and the breakage distribution are decoupled. The breakage rate is related to the probability of particle breakage under an impact, and increases for the weaker particles. As the grinding proceeds, the particles have