Advanced Powder Technology 27 (2016) 1233–1243

Contents lists available at ScienceDirect

Advanced Powder Technology

journal homepage: www.elsevier.com/locate/apt

Original Research Paper

Contact time at impact of spheres on large thin plates

Peter Müller *, Ronny Böttcher, Alexander Russell, Michael Trüe, Sergej Aman, Jürgen Tomas

Mechanical Process Engineering, Otto von Guericke University of Magdeburg, Universitätsplatz 2, 39106 Magdeburg, Germany

ARTICLE INFO

Article history:
Received 24 December 2015
Received in revised form 30 March 2016
Accepted 14 April 2016
Available online 23 April 2016

Keywords:
Coefficient of restitution
Collisions
Contact time
Elastic waves
Impact

ABSTRACT

Using a free-fall apparatus, the low-velocity impact behavior of steel spheres impacting on large thin glass plates had been experimentally measured. The impact and rebound velocities had been determined from the time interval between consecutive impacts and thereby the coefficient of restitution (CoR) had been evaluated. The contact time had been measured from the transmitted current during contact of the sphere (soldered with a very thin flexible conductive wire) and the plate (coated with a thin conductive layer) when a closed circuit is created. The influences of the impact velocity, the sphere size and the plate thickness on the impact behavior had been investigated.

The measurement values had been compared with the theoretical predictions of the Hertz and the Zener models. Thereby, a detailed discussion is presented about the inadequacy of the Hertz model to accurately predict the contact time (in comparison to the better applicable Zener model) for a sphere impacting on an infinitely extended elastic plate of finite thickness, where longitudinal and transversal waves can propagate and travel several times (below the contact area) through the plate within the contact time whereby flexural waves arise. Similarly, the limitations of the Zener model such as the overestimations of the CoR (due of negligence of energy dissipation by friction and viscous damping) are also illustrated with help of experimental results and thereby, the effective limits of reliability of the Zener model are discussed considering the underestimations of the contact time for thin plates.

1. Introduction

In the year 1881, Heinrich Hertz [1] published a theory about the mechanics of elastic isotropic spherical bodies (of diameter d) contacting each other at a very small part of their surfaces (forming a circular contact area of diameter $d_K \ll d$) and mutually exerting a finite pressure on each other (through the contact), thereby causing a perfectly elastic contact deformation (i.e. particle center approach). This theory of Hertz (valid for linear elasticity) has been traditionally used to study the impact behavior of spherical elastic bodies due to its simplicity and high accuracy in predicting the non-adhesive elastic contact deformation.

Considering the decreasing velocity of the bodies with increasing contact force and contact deformation at each instant during (the period of) impact, both bodies would reach an apparently static *equilibrium state* when the elastic contact deformation work done equals the kinetic energy of impact. Since, this equilibrium state corresponds to apparently static conditions, the contact pressure and the contact deformation can be accurately evaluated using known relations to study mechanics of bodies at rest.

Furthermore, Hertz [1] assumed that the contact time t_{el} is large in comparison to the propagation time of elastic waves t_{wave} through the total dimension of the impacting bodies ($t_{el} \gg t_{wave}$). When one considers the Hertzian theory for the case of a rigid sphere impacting on a large and stiff rigid plate, one would find that with the exception of the apparently static equilibrium state, the impacting sphere as well as the plate behave as solid rigid bodies throughout the period of impact. Hence, a dimensionless critical contact time $t_{crit,1}$ consisting of the contact time t_{el} related to the propagation time of elastic waves t_{wave} can be determined as [2]

$$t_{crit,1} = \frac{t_{el}}{t_{wave}} \gg 1. \tag{1}$$

Moreover, since Hertz's derivations are based upon the assumption that the impacting bodies are of infinite dimensions, the theory remains valid only when the elastic wave propagates *just once* through the impact plate. This condition does not remain valid when the impact plate is thin, whereby the elastic waves may travel to-and-fro several times through the plate thickness within the contact time. Thus, when elastic waves which upon reflection at the

* Corresponding author. Tel.: +49 391 67 51886.
E-mail addresses: peter.mueller@ovgu.de (P. Müller), ronny.boettcher@st.ovgu.de (R. Böttcher), alexander.russell@ovgu.de (A. Russell), michael.truee@st.ovgu.de (M. Trüe), sergej.aman@ovgu.de (S. Aman), juergen.tomas@ovgu.de (J. Tomas).

http://dx.doi.org/10.1016/j.apt.2016.04.011
0921-8831/

Nomenclature

Symbols

B	plate width (m)
d	sphere diameter (mm)
c	propagation velocity of elastic waves (m/s)
c'	propagation velocity of quasi-longitudinal waves in thin plates (m/s)
E	modulus of elasticity (GPa)
E_{kin}	kinetic energy of impact (J)
E_{def}	contact deformation energy (J)
e	coefficient of restitution (CoR) (–)
F	force (N)
G	shear modulus (GPa)
H, h	height, thickness (m)
k_{Hz}	Hertzian elastic stiffness constant (GPa mm$^{1/2}$)
L	length (m)
m	mass (g)
n_{refl}	number of wave reflections within the plate (–)
R	radius of surface curvature prior contact flattening (mm)
s	contact displacement (mm)
T	characteristic time constant (s)
t	time (s)
v	velocity (m/s)
α	constant (s/kg)
γ	ratio of sphere diameter to plate thickness (–)
ε	porosity (–)
λ	inelasticity parameter (–)
ν	Poisson's ratio (–)
ρ	density (kg/m^3)
σ	dimensionless displacement (–)
τ	dimensionless time (–)
ψ	sphericity (–)

Indices

A	impact
C	contact
coat	coating
def	deformation
el	elastic
F	plastic contact yielding
fit	adaption
Hz	Hertz
k	sphere
K	contact
kin	kinetic
crit	critical
L	longitudinal wave
loss	loss
max	maximum
min	minimum
P	plate
refl	reflections
RW	Rayleigh wave
s	solid
tot	total
Tr	transverse wave
wave	elastic wave
Z	Zener
1, 2	contact partner 1 (sphere) and 2 (plate)
50	50% quantil of cumulative size distribution

boundary of the plate (i.e. at the reverse plate surface) revert to or travel repeatedly through the region of contact within the contact time, the Hertzian theory cannot accurately predict the contact time. Thus, for the first reverting elastic wave[1], a second dimensionless critical contact time $t_{\mathrm{crit},2}$

$$t_{\mathrm{crit},2} = \frac{t_{\mathrm{el}}}{2t_{\mathrm{wave}}} < 1 \tag{2}$$

and the correspondingly related minimum plate thickness $H_{\min}$

$$H_{\min} = \frac{t_{\mathrm{el}}\, v_{\mathrm{wave}}}{2} \tag{3}$$

can be calculated by given wave propagation velocity v_{wave}. Besides, the dimensionless critical contact time corresponds to the number (frequency) of wave reflections within the plate during contact ($t_{\mathrm{crit},2} = n_{\mathrm{refl}}$).

Summarizing the preceding discussion, during impact, a pressure p develops at the contact area and stress waves arise due to the local contact deformation and propagate inwards through the bodies away from the point of excitation (located at the center of the contact area). Moreover, the generation of surface and body seismic waves produces an energy loss in the contact region. The elastic waves propagate through the solid bodies exhibiting a characteristic velocity c, which are refracted or reflected at interfaces and may propagate back to the point of excitation, where they cause additional energy dissipation. Thus, multiple wave reflections may increasingly affect the energy dissipation associated with the impact event. According to experimental results and theoretical models of several authors [9–13], a considerable contribution of the kinetic energy of impact can be transformed into elastic waves propagating through the solid bodies (see also [8]). Owing to such a practical and fundamental problem, in this paper, we progress toward a systematic solution by evaluating as a first step - the influences of impact velocity, sphere size and plate thickness on the coefficient of restitution (CoR) and the contact time of spheres impacting on relatively large thin plates.

2. Impact mechanics

2.1. The Hertzian approach

The propagation velocity of longitudinal waves (L-waves) c_{L} in isotropic infinite bodies (extended in all sides over many wavelengths) is given by

$$c_{\mathrm{L}} = \sqrt{\frac{E}{\rho}\frac{(1-\nu)}{(1+\nu)(1-2\nu)}} \tag{4}$$

with E the modulus of elasticity, ρ the density and ν the Poisson's ratio, whereas the propagation velocity of transversal waves (Tr-waves) c_{Tr} is given by [3]

$$c_{\mathrm{Tr}} = \sqrt{\frac{G}{\rho}} \tag{5}$$

with G the shear modulus which can be calculated according to

[1] An elastic wave propagating through the contact region during its return upon reflection at the reverse plate surface after travelling once through the whole plate thickness.