



Original Research Paper

On three-dimensional boundary layer flow of Sisko nanofluid with magnetic field effects

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ABSTRACT

This article models the effects of magnetic field and nanoparticles in the three-dimensional flow of Sisko fluid. The flow is caused by a bidirectional stretching surface. Effects of Brownian motion and thermophoresis in the nanofluid model are considered. Sisko fluid is assumed electrically conducted through a constant applied magnetic field. Mathematical formulation in boundary layer regime is presented for a low magnetic Reynolds number. Newly constructed boundary condition subject to zero nanoparticles mass flux at the surface is employed. Nonlinear differential systems are solved for the convergent solutions. Effects of various physical parameters are studied and discussed. Numerical values of skin friction coefficients and Nusselt number are tabulated and analyzed. It is observed that the effects of Brownian motion and thermophoresis parameters on the nanoparticles concentration distribution are quite opposite. Further the temperature and nanoparticles concentration distributions are enhanced for the larger values of magnetic parameter.

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1. Introduction

Extensive studies in the literature deal with the viscous materials that are developed by the classical Navier–Stokes equations. There are many complex rheological materials like polymeric liquids, drilling muds, shampoos, biological fluids, liquid crystals, lubricating oils and many others that show the viscoelastic behavior. These materials cannot be characterized by simple Newtonian model. This situation happens for example when the viscosity index of polymers is improved by submerging them into lubricating oils and makes them less temperatures. In such conditions, these oils show the non-Newtonian behavior. It is all due to wide range usage of lubricants along with its mathematical simplicity that the Sisko fluid model is under investigation in the present work. This fluid model is preferred in view of shear thinning and shear thickening properties. Further Sisko fluid can depict many typical properties of Newtonian and non-Newtonian liquids through selection of the various material parameters. With this viewpoint Sajid and Hayat [1] presented a study to investigate the wire coating analysis of Sisko fluid with drawl from a bath.

Wang et al. [2] analyzed the peristaltic motion of Sisko fluid in both symmetric and asymmetric channels. Molati et al. [3] addressed the unidirectional unsteady flow of Sisko fluid induced by a suddenly moved plate. Time-dependent rotating flow of Sisko fluid in the presence of an applied magnetic field is explored by Abelman et al. [4]. Akyildiz et al. [5] provided the numerical solutions of thin film flow of Sisko liquid past a moving belt. Mathematical model of time-dependent flow of Sisko fluid in the presence of time-variant overlapping stenosis is developed by Mekheimer and El Kot [6]. Ali et al. [7] discussed the unsteady flow of Sisko fluid through a tapered stenotic artery. Recently Munir et al. [8] addressed the bidirectional flow of Sisko fluid due to a stretching surface.

At present the most important need of industrial technologies is ultra-high performance cooling but the low thermal conductivity is the key issue to obtain such ultra-high performance cooling. Thus the nanofluids are considered as the best source to enhance the thermal conductivity of ordinary base fluids. Hence the nanofluids alter the thermal conductivity of ordinary base fluids but the nanoparticles may also alter their thermophysical properties like dynamic viscosity, density and heat capacity. There are also other parameters including Brownian motion and thermophoresis which tend to move the nanoparticles in ordinary base fluids. The nanofluids are utilized in the cooling applications such as electron-

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ics cooling, transformer cooling, vehicle cooling, and silicon mirror cooling. The nanofluids are used to develop the best quality lubricants and oils. The nanofluids are further involved in industrial technologies like food and drink, paper and printing, oil and gas and many others. Especially the magneto nanofluids are useful in MHD pumps and accelerators, MHD power generators, cancer tumor treatment, removal of blockage in the arteries, hyperthermia, wound treatment, sterilized devices, magnetic resonance imaging, etc. Choi [9] investigated that the addition of nanoparticles in the base liquids enhances the thermal conductivity of the base liquids. After Choi [9], Buongiorno [10] developed a mathematical model of nanofluids which includes the Brownian motion and thermophoresis effects. Khan and Pop [11] employed the Buongiorno model [10] to study the boundary layer flow of nanofluid over a linear stretching surface. Literature on flows of nanofluids is quite sizeable. However, some attempts in this direction may be mentioned through the works [12–30] and several studies therein.

The objective of the present communication is to generalize the research of study [8] into three directions. Firstly to consider the flow analysis in the presence of nanoparticles. Attention has been mainly focused to the Brownian motion and thermophoresis. Secondly to utilize the newly proposed condition with the zero nanoparticles mass flux at the boundary. Thirdly to examine the impact of applied magnetic field under small magnetic Reynolds number. The governing nonlinear mathematical model is computed using the optimal homotopy analysis method (OHAM). The impacts of different parameters on non-dimensional temperature and nanoparticles concentration fields are plotted and analyzed. The values of skin friction coefficients and Nusselt number are computed and discussed through numerical data.

2. Mathematical formulation

Consider the laminar, steady three-dimensional (3D) flow of an incompressible Sisko nanofluid over a bidirectional stretching surface. The Sisko fluid is considered electrically conducting in the presence of constant magnetic field B_0 applied in the z -direction (see Fig. 1). In addition the Hall and electric field effects are neglected. The induced magnetic field is not considered for a small magnetic Reynolds number. Brownian motion and thermophoresis effects are also present. The Cartesian coordinate system is adopted in such a manner that x - and y -axes are taken along the stretching surface and z -axis is perpendicular to it. Let U_w and V_w denote the surface stretching velocities along the x - and y -directions respectively. The thermophysical properties of fluid are taken to be constant. Further, we will treat the nanofluid as a

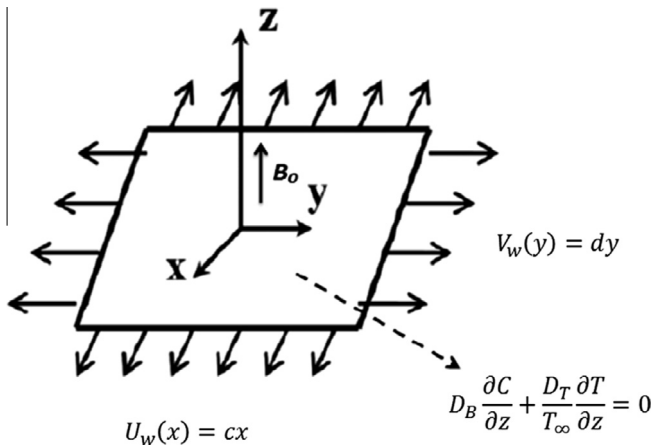


Fig. 1. Geometry of the problem.

two-component mixture (base fluid + nanoparticles) with the following assumptions:

- (1) Incompressible flow
- (2) No chemical reactions
- (3) Negligible external forces
- (4) Dilute mixture ($C \ll 1$)
- (5) Negligible viscous dissipation
- (6) Negligible radiative heat transfer
- (7) Nanoparticles and base fluid are locally in thermal equilibrium.

The constitutive equations for the three-dimensional (3D) flow of an incompressible Sisko fluid are

$$\text{div } \mathbf{V} = 0, \quad (1)$$

$$\rho \frac{d\mathbf{V}}{dt} = \text{div } \boldsymbol{\tau} + \rho \mathbf{b}. \quad (2)$$

The Cauchy stress tensor $\boldsymbol{\tau}$ for Sisko fluid is

$$\boldsymbol{\tau} = -p^* \mathbf{I} + \mathbf{S}, \quad (3)$$

where \mathbf{S} is the extra stress tensor given by

$$\mathbf{S} = \left[a + b \left| \sqrt{\frac{1}{2} \text{tr}(\mathbf{A}_1^2)} \right|^{n-1} \right] \mathbf{A}_1, \quad (4)$$

in which a , b and n ($n \geq 0$) are the material constants of the Sisko fluid, p^* is the pressure and \mathbf{b} is the body force. The first Rivlin-Erickson tensor \mathbf{A}_1 is defined as follows:

$$\mathbf{A}_1 = \text{grad } \mathbf{V} + (\text{grad } \mathbf{V})^*, \quad (5)$$

where $*$ represents the matrix transpose and the velocity field \mathbf{V} is given by

$$\mathbf{V} = [u(x, y, z), v(x, y, z), w(x, y, z)]. \quad (6)$$

With the boundary layer assumptions in Ref. [31], the governing boundary layer expressions in the Sisko nanofluid with heat and mass transfer can be expressed as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (7)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{a}{\rho_f} \frac{\partial^2 u}{\partial z^2} - \frac{b}{\rho_f} \frac{\partial}{\partial z} \left(-\frac{\partial u}{\partial z} \right)^n - \frac{\sigma B_0^2}{\rho_f} u, \quad (8)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \frac{a}{\rho_f} \frac{\partial^2 v}{\partial z^2} + \frac{b}{\rho_f} \frac{\partial}{\partial z} \left(-\frac{\partial u}{\partial z} \right)^{n-1} \frac{\partial v}{\partial z} - \frac{\sigma B_0^2}{\rho_f} v, \quad (9)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_m \frac{\partial^2 T}{\partial z^2} + \frac{(\rho c)_p}{(\rho c)_f} \left(D_B \left(\frac{\partial T}{\partial z} \frac{\partial C}{\partial z} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial z} \right)^2 \right), \quad (10)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_B \left(\frac{\partial^2 C}{\partial z^2} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial z^2} \right). \quad (11)$$

The subjected boundary conditions are

$$u = U_w(x) = cx, \quad v = V_w(y) = dy, \quad w = 0, \quad T = T_w, \\ D_B \frac{\partial C}{\partial z} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \quad (12)$$

$$u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ as } z \rightarrow \infty. \quad (13)$$

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