Advanced Powder Technology 26 (2015) 1191–1199

Contents lists available at ScienceDirect

Advanced Powder Technology

journal homepage: www.elsevier.com/locate/apt

Original Research Paper

# Study on free fall surfaces in three-dimensional hopper flows

Y. Tian [a], P. Lin [a], S. Zhang [a,b], C.L. Wang [a], J.F. Wan [a,b], L. Yang [a,*]

[a] Institute of Modern Physics, Chinese Academy of Sciences, 509 Nanchang Road, 730000 Lanzhou, China

[b] University of Chinese Academy of Sciences, No. 19A Yuquan Road, 100049 Beijing, China

ARTICLE INFO

*Article history:*
Received 9 March 2015
Received in revised form 3 May 2015
Accepted 31 May 2015
Available online 9 June 2015

*Keywords:*
Free fall surfaces
DEM
Funnel flow
GPU

ABSTRACT

The flow rate of grains discharging from hoppers can be quantified by Beverloo's law, where the assumption of the free fall arch (FFA) it is very useful in understanding the physical picture of this process but is difficult to be observed directly. Here simulations of 2,000,000 spheres discharging from three-dimensional flat-bottomed hopper with circular outlet were performed on multiple GPUs and the free fall surfaces were explored by statistical analysis. The free fall surfaces in hopper flows were plotted and can be fitted into parabolic surfaces. Other quantities, such as velocity fields and spatial profiles of coordination number, were investigated to support the free fall surfaces we obtained. Although inconsistent with the initial assumption, it is shown that the free fall surfaces can be statistically described and this work provides new insights for understanding the processes of hopper flows.



## 1. Introduction

Granular materials discharging from hoppers has been widely studied in the last decades [1–7]. Unlike normal fluids, granular materials discharge constantly by gravity independent of filling height because of Janssen effect [8]. The flow rate is also independent of the hopper width and height when they are larger than 2.5 times of the outlet diameter [6]. Based on the experimental/empirical knowledge, the concept of an arch structure below which the grains fall freely was firstly introduced by Hagen in the mid-19th century [9,10]. In 1961, Brown developed a 'minimum energy theorem' to describe granular flow which falls freely under gravity near apertures [11], and later Brown and Richards photographed the free fall region above the outlet of a two-dimensional (2D) rectangular frame in hopper flows of ball bearings. They named the boundary of this region as 'free fall arch' [12,13]. Le Pennec et al. found the FFA existing in ticking hour glasses when the flow is intermittent likewise [14]. Barletta et al. applied the FFA in two-phase flow of gas and grains [15]. Furthermore, the FFA theory was applied to various experiments, such as funnel flows and mass flows in conical hoppers [16] and tilted hoppers [17]. In recent years, the FFA was also considered in several computer simulations of granular flows with various simulation methods [2,5,18–20]. Lately Vivanco's experiment, which used photo-elastic plates in wedge hoppers, suggested that there were no permanent free fall arches during the flow but a time-averaged of the network of contact forces could show a boundary with characteristics resembling the FFA when the outlet was not large [21]. Because the characteristics of the FFA are hardly studied by direct observation in experiments, the detailed physical picture of the FFA is still unclear especially in three-dimensional (3D) hoppers [22], and the relationship between the FFA and parameters in various granular flows is obscure.

The widely accepted formula of flow rates called 'Beverloo's law' was proposed by Beverloo et al. in 1961 [1]. This formula has a unified form for various types of hoppers: $\varphi = C\rho_f\sqrt{g}RA$, where $\varphi$ is the flow rate, $C$ is a dimensionless coefficient, $R$ is the equivalent hydrodynamic radius, $\rho_f$ is the equivalent density at the outlet, $g$ is the acceleration of gravity and $A$ is the equivalent hydrodynamic section. It is valid when $R$ is much larger than the particle's size [23,24]. Particularly for a 2D hopper, $R = 0.5(D_0 - kd)$ and $A = D_0 - kd$ where $D_0$ is the width of the outlet, $k$ is a dimensionless coefficient and $d$ is the particle diameter. In this case the law can be rewritten as $\varphi = C\rho_f\sqrt{g}(D_0 - kd)^{1.5}$ [19]. Similarly for 3D hoppers with round outlets, it is $\varphi = C\rho_f\sqrt{g}(D_0 - kd)^{2.5}$ [1] and for hoppers with slot outlets, it is $\varphi = C\rho_f\sqrt{g}(L_x - kd)(L_y - kd)^{1.5}$ [19,25,26], where $L_x$ is the long side length of the slot and $L_y$ is the short one. The FFA [2,16,20] which was assumed to exist above the outlet is useful for understanding the law. In this assumption, the velocities of grains above the arch can be neglected and grains below the arch fall freely under gravity. If the height of the FFA is proportional to the outlet

* Corresponding author. Tel.: +86 9314969187; fax: +86 9314969189.
*E-mail address:* lyang@impcas.ac.cn (L. Yang).

http://dx.doi.org/10.1016/j.apt.2015.05.015
0921-8831/

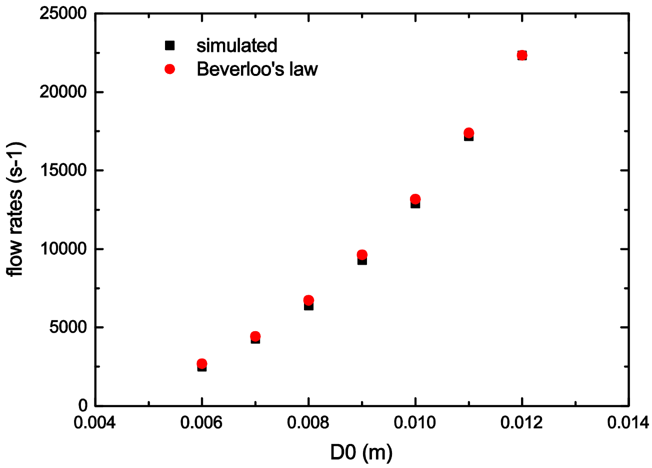


**Fig. 1.** Discharge rate versus outlet diameter.

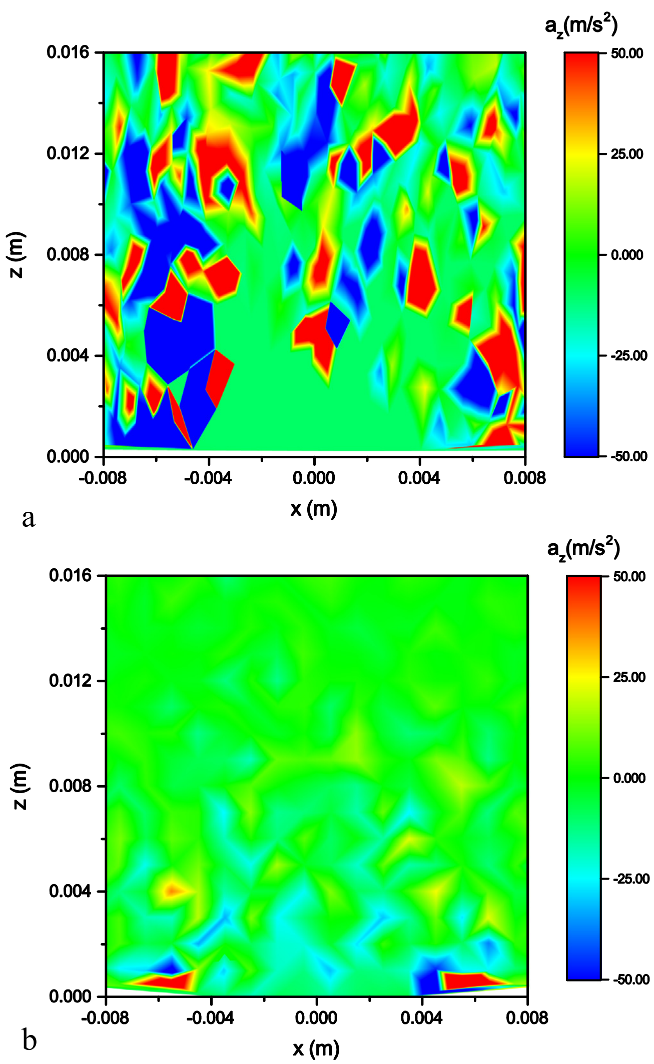


**Fig. 2.** Profiles of vertical accelerations on the cross section through the symmetry axis with $D_0 = 10\ d$ and $d = 0.001$ m. (a) A snapshot at 1 s. (b) A time-averaged result in 1–4 s after the flows begins.

**Table 1**
Parameters of steel spheres in our simulations.

| Quantity | Symbol | Value |
|---|---|---|
| Elastic modulus (GPa) | $E$ | 206 |
| Shear modulus (GPa) | $G$ | 79 |
| Poisson's ratio | $v$ | 0.25 |
| Friction of spheres | $\mu$ | 0.20 |
| Diameter of spheres (m) | $d$ | 0.005 |
| Density of spheres (kg/m$^3$) | $P$ | 7850 |
| Coefficient of restitution | $\varepsilon$ | 0.95 |

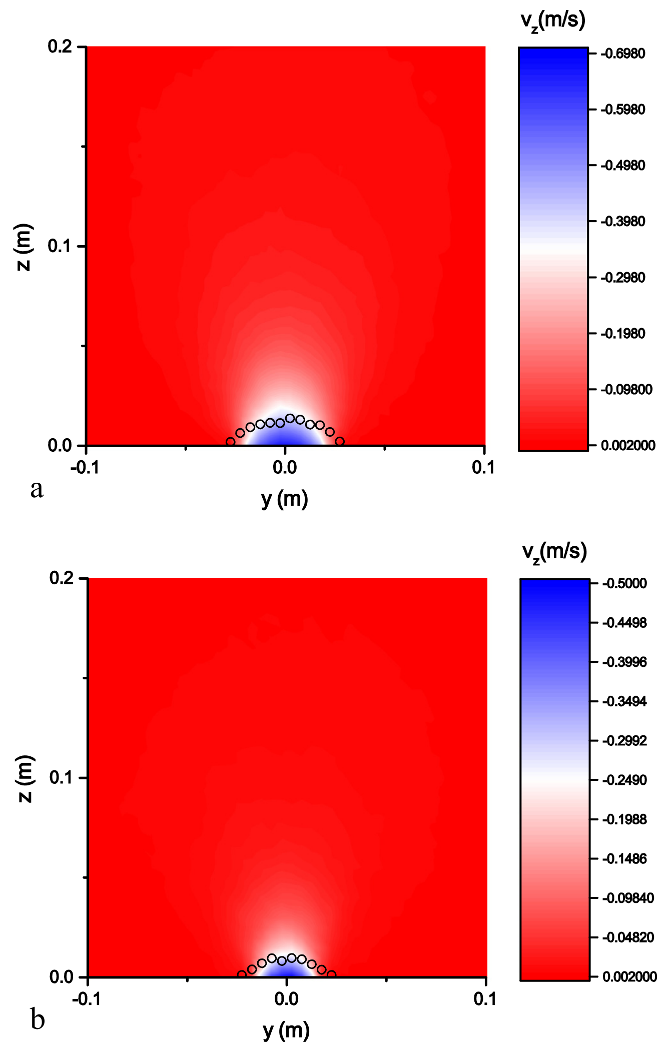


**Fig. 3.** Vertical velocity on the section across the symmetry axis (a) $D_0 = 10\ d$; and (b) $D_0 = 7\ d$. The dots are the free fall surfaces from our calculation.

diameter, the grains velocities at the outlet should be proportional to $\sqrt{gR}$. With $A$ being the section the grains flow through, the output volumetric flow rate should be should be $\sqrt{gR}A$ and the flow rate should be proportional to $\rho_f\sqrt{gR}A$. As noted above, the assumption of the FFA in the hopper flow provides a good explanation for the scaling of Beverloo's law.

In this work, dense granular flows were studied in the 3D flat-bottomed hoppers by discrete element method (DEM) simulations. Various physical quantities in the hopper flows, such as spatial profiles of velocities, the fraction of grains with acceleration of gravity and coordination number of in the funnel flows near the outlet were investigated statistically. By analyzing simulation results of these physical quantities, the statistical description of the shapes of the arch structures were presented and compared with the ideal FFA. Additionally the test of the free fall by velocities and the crosscheck of flow rates are performed. In contrast to a