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A plasticity-corrected stress intensity factor for fatigue crack growth in ductile materials

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Abstract

In this paper, a plasticity-corrected stress intensity factor range ΔK_{pc} is developed on the basis of plastic zone toughening theory. Using this new mechanical driving force parameter for fatigue crack growth (FCG), a theoretical correlation of Paris's law with the crack tip plastic zone is established. Thus, some of the important phenomena associated with the plastic zone around the fatigue crack tip, such as the effects of load ratio R, overload and T stress on the FCG behavior, can be incorporated into the classical Paris's law. Comparisons with the experimental data demonstrate that ΔK_{pc} as a single and effective mechanical parameter is capable of describing the effects of the load ratio, T stress and overload on the FCG rate. The FCG rate described as a function of ΔK_{pc} tested under a simple loading condition can also be used for other complex loading conditions of the same material.

Keywords: Fatigue crack growth; Plastic zone; Stress intensity factor; Ductile fracture

1. Introduction

Fatigue fracture is known to be one of the main failure modes in engineering components. Although fatigue has been the subject of thorough studies for many decades, it still remains a highly topical research direction for mechanical engineering and materials science. Linear elastic fracture mechanics has been a powerful tool for dealing with fatigue crack growth (FCG) for many decades. Paris et al. [1,2] postulated that the crack advance per cycle, $\mathrm{d}a/\mathrm{d}N$ (where a is the crack length, and N is the number of cycles), depends on the applied elastic stress intensity factor (SIF) range ΔK_{el}, via a power law

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K_{el})^m \tag{1}$$

where $\Delta K_{el} = K_{el}^{max} - K_{el}^{min}$ and K_{el}^{max} and K_{el}^{min} are, respectively, the SIF of the crack at the maximum and minimum applied loads (or stresses) per cycle, denoted by S_{max} and S_{min}. C and m are the material parameters found from experiments.

Paris's law was widely accepted as the principal basis for the FCG rate predictions for decades. In theory, Paris's law holds only when the crack propagation is controlled by elastic deformation occurring around the crack tip. However, plastic deformation is almost always present in the vicinity of a stressed crack tip for ductile metal materials. The fatigue fracture is significantly affected by the plastically deformed zone around the crack tip [3–5]. When the plastic zone occurs around the crack tip, Paris's law suffers from two major drawbacks in its application to ductile fracture: (1) the lack of explicit dependence on the maximum load or load ratio $R = S_{min}/S_{max}$ (which controls the extent of plasticity within each cycle); and (2) the lack of dependence on the loading history [6].

It is widely assumed that, under small scale yielding (SSY) conditions, the K field still controls the fatigue crack growth. On the basis of this assumption, many attempts have been made to account for the influence of the plastic zone. Elber [7] discovered the so-called crack closure

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behavior induced by the crack-tip plastic zone and suggested a new parameter, a range of effective stress intensity, $\Delta K_{\mathrm{eff}} = \Delta K_{\mathrm{el}}(S_{\max} - S_{\mathrm{op}})/(S_{\max} - S_{\min})$, where S_{op} is the opening load (or stress). Elber's pioneering work has then been enhanced by many researchers, and it has been widely used as a critical mechanism responsible for the plastic zone effect [8–11]. However, the crack closure-based approaches remain the subject of ongoing argument. In particular, it has been reported that different measurement techniques result in different closure magnitudes [6]. Some additional difficulties associated with the crack closure methodology have been discussed by Dinda and Kujawski [12], Sadananda and Vasudevan [13] and Kujawski [14].

Therefore, attempts have also been made to reformulate Paris's law with alternative methods, such as by combining the maximum elastic SIF $K_{\mathrm{el}}^{\max}$ and the SIF range ΔK_{el} [12,14,15], relating $K_{\mathrm{el}}^{\max}$ (or ΔK_{el}) to crack-tip blunting [9–11], to the crack-tip plastic zone size [16–18] or to the energy dissipation in the vicinity of the crack tip [19,20]. These methods have brought, from different aspects, much better understanding to the fatigue crack behavior. However, as crack closure-based approaches, they were strictly empirical. Besides their own limitations in practical application, none of the above methods can capture the influence of the load history on FCG.

In spite of the important role that the plastic zone can play in FCG, to date there has not been a well-recognized method of correlating Paris's law with crack-tip plastic deformation. The key reason is that a sound mechanical model has not yet been established for quantitative evaluation of the effects of the crack-tip plastic zone on the SIF.

Recently, Li and co-authors [21–23] demonstrated that a plastically deformed zone around a stressed crack tip can be identified with an inclusion of transformation strain by means of Eshelby's equivalent inclusion method. The toughening effect of the plastic zone on the crack can then be evaluated quantitatively by the present transformation toughening theory. According to this plastic zone toughening theory, the closed-form solution for change in SIF due to the crack-tip plastic zone is given for both the mode I and mode II cracks under SSY conditions [22,23].

Thus, the objective of this study is to demonstrate that, for a fatigue crack, this closed-form solution can be used to develop a plasticity-corrected (PC) SIF range, which can directly correlate Paris's law with the crack-tip plastic deformation zone. The application of the PC-SIF range to fatigue loading will be illustrated by manifesting the effects of load ratio, overload and T stress on the FCG rate.

2. The PC-SIF range ΔK_{pc}

According to Eshelby's inhomogeneity theory [24], an inhomogeneity in materials can be transformed to a homogeneous one with the transformation strain. Thus, the crack–inhomogeneity interaction can be determined based on Hutchinson's general solution for the crack–transformation strain interaction [25]. In a previous study [21], it was theoretically proved and numerically verified that a plastic zone can be equivalent to a homogeneous inclusion with transformation strain. Thus, Eshelby's inhomogeneity theory and Hutchinson's solution provide a sound basis for assessing the effect of the crack-tip plastic zone on the SIF. Recently, an analytical solution for change in mode I SIF due to the crack-tip plastic zone was given by

$$\Delta K_{\mathrm{pl}} = \frac{1}{2\sqrt{2\pi}} \int_{\Omega} r^{-3/2} \left[\frac{K_{\mathrm{el}}}{\sqrt{2\pi r}} \left(2\cos\frac{\theta}{2}\cos\frac{3\theta}{2} + 3\sin^2\theta\cos\theta \right) + 3(\sigma_{11} - \sigma_{22})\sin\theta\sin\frac{5\theta}{2} - 6\sigma_{12}\sin\theta\cos\frac{5\theta}{2} - (\sigma_{11} + \sigma_{22})\cos\frac{3\theta}{2} \right] d\Omega \tag{2}$$

for the plane strain and SSY conditions [22,26]. In Eq. (2), K_{el} is the remotely applied elastic SIF, Ω is the area of the plastic zone around the crack tip, and σ_{11}, σ_{22}, σ_{12} are the stresses in the plastic zone. Eq. (2) indicates that, if the distributions of the stresses in the plastic zone as well as the shape and size of the plastic zone are known, the change in the SIF due to the plastic deformation can be evaluated. Accordingly, a PC-SIF, ΔK_{pc}, can be defined as

$$K_{\mathrm{pc}} = K_{\mathrm{el}} + \Delta K_{\mathrm{pl}} \tag{3}$$

The numerical results [22] showed that the crack-tip plastic zone has a significant shielding effect on the crack-tip field, i.e., a negative value of ΔK_{pl} was found for stationary cracks.

Eq. (2) was developed using Eshelby's equivalent inclusion theory. The essence of Eshelby's equivalent inclusion theory is the stress equivalence principle [27]. According to this principle, Li et al. [28] demonstrated that the term "inhomogeneity" in Eshelby's inclusion theory represents the situation where the misfit strain presents in a finite sub-domain Ω within an elastic body. It may be a pore, a gas bubble, a shear band or a plastically deformed zone. Thus, it is expected that Eq. (2) can be applied to the cyclic plastic-zone for a fatigue crack.

For a fatigue crack, the PC-SIF range ΔK_{pc} can be defined as

$$\Delta K_{\mathrm{pc}} = K_{\mathrm{pc}}^{\max} - K_{\mathrm{pc}}^{\min} \tag{4}$$

where $K_{\mathrm{pc}}^{\max}$ and $K_{\mathrm{pc}}^{\min}$ are the PC-SIF associated with the maximum and minimum loads in a loading cycle according to Eq. (3). Formally, Eq. (4) is applicable for FCG under tensile–tensile loading (load ratio $R > 0$) because Eq. (2) is derived under tensile loading where the SIF concept has a definite meaning. However, it is well known that the compressive part of cyclic loading, under tensile–compression loading (load ratio $R < 0$), has a substantial influence on the FCG behavior [29]. This is attributed to the plastic deformation of the material produced during compressive loading. According to ASTM-E647, the compression part of loading is, however, neglected in calculation of the SIF range used in Paris's law. This is because one assumes that the fracture surfaces are in contact during