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Partitioning of elastic energy in open-cell foams under finite deformations

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Abstract

The challenges associated with the computational modeling and simulation of solid foams are threefold—namely, the proper representation of an intricate geometry, the capability to accurately describe large deformations, and the extremely arduous numerical detection and enforcement of self-contact during crushing. The focus of this study is to assess and accurately quantify the effects of geometric nonlinearities (i.e. finite deformations, work produced under buckling-type motions) on the predicted mechanical response of open-cell foams of aluminum and polyurethane prior to the onset of plasticity and contact. Beam elements endowed with three-dimensional finite deformation kinematics are used to represent the foam ligaments. Ligament cross-sections are discretized through a fiber-based formulation that provides accurate information regarding the onset of plasticity, given the uniaxial yield stress–strain data for the bulk material. It is shown that the (hyper-) elastic energy partition within ligaments is significantly influenced by kinematic nonlinearities, which frequently cause strong coupling between the axial, bending, shear and torsional deformation modes. This deformation mode-coupling is uniquely obtained as a result of evaluating equilibrium in the deformed configuration, and is undetectable when small deformations are assumed. The relationship between the foam topology and energy partitioning at various stages of moderate deformation is also investigated. Coupled deformation modes are shown to play an important role, especially in perturbed Kelvin structures where over 70% of the energy is stored in coupled axial-shear and axial-bending modes. The results from this study indicate that it may not always be possible to accurately simulate the onset of plasticity (and the response beyond this regime) if finite deformation kinematics are neglected.

Keywords: Open-cell foams; Cellular structures; Finite deformation; Energy partition; Fiber-based models

1. Introduction

Solid foams are an increasingly important class of materials. Their distinctive microstructure, ubiquitous in nature, leads to unique properties including extraordinarily high stiffness-to-mass ratio and energy absorption capacity [1,2]. Numerical simulations of the mechanical response of these materials are particularly difficult; and complications include the generation of a geometry that accurately replicates their intricate microstructure [3–5], the broad range of strains that foams can undergo, and the extreme computational challenge involved with the simulation of the densification process [6,7].

The objective of the present study is to evaluate the influence of kinematic nonlinearities prior to the onset of plasticity in metallic and polymeric foam ligaments on the distribution of stored energy in different modes of deformation (i.e. torsion, bending, stretching, etc.) in response to an applied external load. The present study is limited to deformations up to the onset of plasticity in foam ligaments, and excludes ligament-to-ligament contact, or fracture. Consideration of finite deformation kinematics at the ligament scale provides a more accurate distribution of the internal strain energy within various modes of deformation. Understanding energy distribution should provide an insight into how loads are transmitted

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though the network of ligaments, and the discovery of internal load/stress paths in ligaments up to the point of initiation of plasticity under applied macroscale loading.

A primary incentive for understanding the dominant deformation modes and load-sharing behavior in open-cell foam ligaments is to identify—or even to design—specific topologies that are ideally suited to particular application needs. Prior related studies include those by Hutchinson and Fleck [8] and Deshpande et al. [9] who first investigated the two primary modes of deformation, and compared the responses of bending- vs. stretching-dominated structures. They showed that stretching-dominated structures tend to be stronger and stiffer than bending-dominated ones, and determined the minimum nodal connectivity required to obtain a stretching-dominated structure for a special class of lattice structures. More recently, Alkhader and Vural [10] employed Timoshenko beams to explore the distribution of contributing modes of deformation (pure-bending and pure-axial, only) for three different specimens with different nodal connectivities.

In the present study, we aim to obtain an accurate understanding of open-cell foam response prior to the onset of plasticity. The difficulties lie in the proper description of the large deformations of foam ligaments and the development of an accurate procedure to detect the initiation of plastification through members. In contrast to continuum finite elements, the use of beam elements allows us not only to gain significant computational speed, but also, to directly determine stress and strain resultants, which, in turn, facilitates a straightforward partitioning of the internal energy. We attempt to achieve this objective by analyzing three-dimensional open-cell foam specimens with varying attributes of nodal connectivity, morphological regularity and material type. We model the geometric attributes of each ligament in requisite detail, and partition the elastic strain energy into 10 modes of deformation as offered by the Timoshenko beam formulation with finite deformation kinematics—namely, four pure modes of deformation (i.e. axial, bending, shear and torsional) along with the six corresponding coupled modes.

The remainder of the manuscript is organized as follows: the procedure adopted to construct the ligament and representative volume element (RVE) geometries is presented in Section 2. The salient features of finite deformation kinematics for three-dimensional Timoshenko beams, and a fiber-based discretization technique used for detecting first-yield are provided in Section 3. The effects of geometric nonlinearities of single-foam ligaments are quantified in Section 4. An approach for decomposing the internal ligament energy into all 10 modes of deformation, along with a recursive formula that accumulates the decomposed energies for an arbitrary loading path, are provided in Section 5. Finally, in Section 6, periodic and perturbed foam topologies with different nodal connectivities are analyzed in order to assess the relative importance of each mode of deformation in metallic and polymeric foams.

2. Topologies under investigation

The topologies of cellular solids depend on the physical processes they undergo during manufacturing [11]. Different manufacturing routes yield specimens with notable differences in cell shape, size and type, which, in turn, impact their mechanical behavior. It has long been recognized that there is a strong similarity between many foaming processes and the mathematical construction of Voronoi cells (see e.g. [12–14]). If we consider a set of seed points randomly populating a given space, then a Voronoi tessellation associates each of the seed points to a surrounding region in space, which is the set of all points closer to this seed point than any other seed point. A convex polygon partition of the space is obtained by repeating this operation for every seed point [15,16]. This method can be compared to the idealized growth of bubbles in a liquid. If the growth rate of each individual bubble were identical, then the final geometry would be that of a Voronoi tessellation. Multitudes of cell types can be created by positioning a specific set of seed points in space and by applying the Voronoi algorithm to generate a network of beams in two- or three-dimensional space. The topology and regularity of the final structure only depends on the position of the initial seeds.

2.1. Arrays of cubes

The cube is the simplest space-filling structure. This geometry was used by Ashby et al. [17] as a unit-cell to investigate dimensional laws relating relative density to mechanical properties. It can easily be constructed through a Voronoi tessellation by positioning the seed points in a simple cubic (sc) pattern to obtain the structure shown in Fig. 1 (left).

2.2. Kelvin cells

In this case, the Voronoi seed points are positioned in a body-centered cubic (bcc) pattern throughout the lattices, and will give rise to arrays of tetrakaidecahedra shown in Fig. 1 (middle). These structures—also known as Kelvin cells—have 6 squares and 8 hexagons for a total of 14 faces per cell. The structure is a highly optimized partition of space into cells of fixed equal volume with minimum surface area. As spheres of gaseous alloy grow and interact during the foaming process, Kelvin cells are deemed to naturally appear, because the surface energy is being minimized this way [11].

2.3. Triangular prisms

With recent advances in solid free-form fabrication, any topology that can be imagined can successfully be manufactured. While rarely encountered in practice, cellular structures composed of triangular prisms can nonetheless be manufactured (Fig. 1, right). In the present study, we will investigate them alongside cubic and Kelvin cells, in