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Permeability of dual structured hypoeutectic aluminum alloys

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Abstract

A mathematical expression to describe the evolution of permeability during equiaxed eutectic solidification of hypoeutectic aluminum alloys has been developed by considering the solidifying microstructure to be a dual structured system consisting of a network of equiaxed, dendritic and eutectic grains. The permeability of hypoeutectic aluminum alloy microstructures was characterized on simulated dendritic/eutectic microstructures predicted using a cellular automaton technique starting from a primary dendritic structure characterized via X-ray microtomographic analysis. The permeability was characterized (i) physically using large-scale analogs of the simulated microstructures and (ii) numerically by predicting the flow through the simulated microstructures. The permeability values determined through physical and numerical modeling are in good agreement with each other and are consistent with the mathematical expression. The proposed permeability expression is valid over the complete solidification range and for a wide range of compositions. The expression reduces to the conventional Carman–Kozeny expression during dendritic solidification and/or dendritic/eutectic solidification with a low density of eutectic grains. However, it deviates from the conventional Carman–Kozeny expression as the density of eutectic grains increases.

Keywords: Permeability; Eutectic solidification; Cellular automaton; X-ray microtomography; Aluminum alloys

1. Introduction

Many commercially relevant casting alloys have significant fractions of eutectic mixture in their microstructures. Differences in the size, shape and distribution of the eutectic grains affect the mechanical performance of the cast component. Additionally, the differences in eutectic structure can affect late stage feeding during solidification, leading to porosity formation. For example, Knuutinen et al. [1] showed that there is a strong correlation between the amount and distribution of porosity and the eutectic solidification mode in aluminum alloy A356. With the continued interest and recent advances in modeling casting processes, characterizing the evolution of permeability during eutectic solidification is crucial for accurate defect prediction.

Experimental studies to measure the permeability of different alloys have focused predominantly on samples with equiaxed dendritic microstructures as the primary phase [2–11]. To measure the permeability of the primary phase an apparatus called a permeameter, which applies a pressure gradient to push a eutectic liquid through a partially solid sample in an isothermal environment while measuring the discharge rate, may be used. Although the natural tendency of the microstructure to change during the test is a major source of error in such experiments [12], it is still an effective means to measure permeability. For eutectic solidification this measurement methodology cannot be employed because eutectic solidification is an isothermal transformation, making it impossible to maintain the instantaneous microstructure part way through the eutectic transformation.

Due to the difficulties associated with measuring the permeability of a developing eutectic mixture, a physical modeling technique may provide an alternative means of characterizing it. Despois and Mortensen [13] measured

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the permeability of open pore microcellular materials (open foam) by passing glycerin mixed with water through aluminum open pore foams produced using a replication process. These measurements enabled them to propose a permeability equation for open pore materials. James et al. [14] passed oil through a large-scale physical model of repeating slotted plates to measure permeability. Khajeh and Maijer [15] developed a physical modeling technique to measure the permeability of dendritic structures by passing a glycerin-based solution through large-scale analogs of interdendritic structures produced by rapid prototyping. The results of these studies [13–15] indicate that experiments on physical models are a suitable alternative to permeameter measurements.

Researchers have also determined the permeability of interdendritic channels numerically. A number of researchers have calculated the permeability of two-dimensional (2D) dendritic structures obtained from sections of cast alloys [16,17], while some researchers have determined the permeability of three-dimensional (3-D) microstructures [15,18–22]. Early research on complex 3-D microstructures [18–20] employed domains discretized with uniform cubic elements. Khajeh and Maijer [15] showed that since the permeability is a geometric characteristic of a porous medium that depends on the width and tortuosity of the flow channels, an accurate computational domain is critical for numerical determination of permeability. In particular, using a mesh consisting of uniform cubic elements leads to an artificial increase in the complexity of the flow channels.

Notwithstanding the challenges that exist in accurately characterizing the permeability of different eutectic alloys, a comprehensive equation capable of accurately predicting the permeability of evolving eutectic microstructures is necessary for use in macro-scale casting models. For equiaxed structures the Carman–Kozeny equation is often used to describe the permeability K [23]:

$$K = \frac{(1-f_s)^3}{k_C S_v^2 f_s^2} \tag{1}$$

where f_s is the solid fraction, k_C is a constant, and S_v is the solid/liquid interfacial area per unit volume of solid. Uncertainty continues to exist as to whether the Carman–Kozeny equation is applicable over the complete solidification range (from dendritic phase to dendritic/eutectic mixture) and for a wide variation in microstructure. For example, it has been shown that k_C is equal to $\sim$3 [15] and $\sim$5 [22] for dendritic and granular microstructures, respectively. Thus, the value of k_C may be significantly influenced by the geometry of liquid channels. For the limiting cases where complete equiaxed dendritic or eutectic solidification occurs the morphology of the liquid channels is either dendritic or globular, respectively. However, during equiaxed solidification of hypoeutectic aluminum alloys eutectic grains solidify in an existing dendritic network. Hence, the geometry of liquid channels undergoes a transition from dendritic to globular. During this transition the solidifying microstructure can be regarded as a dual structured porous medium. Liu et al. [24] introduced a mathematical expression for an idealized dual structured porous medium, based on an analytical solution [25] to the Brinkman–Darcy equation, to describe the effective permeability of an array of square impermeable blocks placed in a fluid saturated porous medium. They focused on an equivalent continuum model and proposed a mathematical relation for determining the effective permeability. Sano et al. [26] extended the work of Liu et al. [24] to calculate the effective permeability of a porous medium with obstacles of different sizes. Both studies showed excellent agreement between the permeabilities calculated with the developed mathematical expressions and experimental measurements. These results suggest that mathematical expressions developed based on solutions to the Brinkman–Darcy equation may be applicable to non-idealized dual structured porous media.

In this study the permeability of hypoeutectic aluminum alloys for the complete solidification range (from equiaxed dendritic to equiaxed eutectic in a dendritic network) has been investigated. Assuming Eq. (1) applies up to the start of eutectic solidification, the solidifying dendritic/eutectic microstructure has been treated as a dual structured porous medium consisting of a dendritic network overlaid with eutectic grains. A mathematical expression based on solution of the Brinkman–Darcy equation has been developed that reduces to Eq. (1) in the absence of eutectic grains (i.e. prior to the start of eutectic solidification) and describes the evolution of permeability during eutectic solidification. The validity of the mathematical expression is assessed by comparison with permeabilities determined through physical and numerical modeling. These results are put into perspective by comparison with the conventional Carman–Kozeny expression.

2. Theory

In hypoeutectic aluminum alloys equiaxed eutectic grains grow inside the network established by the preceding primary dendritic phase solidification. This growth sequence causes the geometry of the flow channels to evolve from dendritic to more globular. The solidifying microstructure may be treated as a dual structured porous medium and, based on solution of the Brinkman–Darcy equation considering the dendritic/eutectic microstructure, a mathematical expression can be derived to predict the evolution of permeability.

2.1. Permeability based on the Brinkman–Darcy equation

Considering a steady-state, incompressible, and very slow flow (Re < 1) through a horizontal channel filled with a porous medium of absolute permeability K and liquid fraction f_l, the governing equation for the fully developed flow, namely the Brinkman–Darcy equation, is [24]: