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Novel solution for acceleration motion of a vertically falling spherical particle by HPM–Padé approximant

Mohsen Torabi, Hessameddin Yaghoobi *

Department of Mechanical Engineering, Faculty of Engineering, Semnan University, Semnan, Iran

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ABSTRACT

In this paper the acceleration motion of a vertically falling spherical particle in incompressible Newtonian media is investigated. The velocity is evaluated by using homotopy perturbation method (HPM) and Padé approximant which is an analytical solution technique. The current results are then compared with those derived from HPM and the established fourth order Runge–Kutta method in order to verify the accuracy of the proposed method. It is found that this method can achieve more suitable results in comparison to HPM.

1. Introduction

Non-linear phenomena play a crucial role in applied mathematics and physics. We know that most of engineering problems are non-linear, and it is difficult to solve them analytically. Various powerful mathematical methods have been proposed for obtaining exact and approximate analytic solutions.

Recently, He [1,2] proposed the homotopy perturbation method (HPM) and variational iteration method (VIM) for solving linear, non-linear, initial, and boundary value problems. It is worth mentioning that the origin of variational iteration method can be traced back to Inokuti et al. [3], but the real potential of this technique was explored by He. Moreover, some researchers realized the physical significance of HPM and VIM, its compatibility with the physical problems and applied this promising technique to a wide class of linear and non-linear, ordinary, partial differential equations [4–11].

The problem of describing the accelerated motion of a falling sphere in Newtonian fluids is relevant to many situations of practical interest. Typical examples include unit operations, such as classification, centrifugal and gravity collection and separation, where it is often important to know the detailed trajectories of the accelerating particles for purposes of design or improved operation. In other practical situations, for example raindrop terminal velocity measurements, or viscosity measurements in Newtonian fluids using the falling ball method, it is also necessary to know the time and distance required to reach terminal velocity for a given sphere-fluid combination prior to making the reliable determination of the sphere settling velocity. Owing to the importance of the aforementioned applications, considerable attention has been devoted to the study of the accelerated motion of a sphere in a fluid, and an excellent account of the theoretical developments in this area has been given by Clift et al. [12] for Newtonian fluids. Recently, analytical methods [13–17] have been used to describe the transient motion of the falling sphere and non-sphere in Newtonian fluids. Jalaal and Ganji [13] studied the unsteady motion of a spherical particle rolling down an inclined plane submerged in a Newtonian environment using a drag of the form given by Chhabra and Ferreira [18], for wide range of Reynolds numbers by HPM. Jalaal et al. [14] applied VIM on the acceleration motion of a non-spherical particle in an incompressible Newtonian environment for a wide range of Reynolds numbers using a drag coefficient as defined by Chien [19]. In [4,15] the unsteady motion of a spherical particle falling in a Newtonian fluid was analyzed using HPM. Jalaal et al. [16] analyzed the motion of a spherical particle in a plane couette flow. Jalaal et al. [17] applied homotopy analysis method (HAM) to obtain exact analytical solutions for unsteady motion of a spherical particle rolling down an inclined tube submerged in an incompressible Newtonian environment.

To obtain precise velocity of a falling particle, an accurate relationship between Reynolds numbers and drag coefficient is required. In this work, we study the accelerated motion of a falling spherical particle with a general drag coefficient of form given by

* Corresponding author. Tel.: +98 91 28055461; fax: +98 21 77180590.
E-mail address: Yaghoobi.Hessam@gmail.com (H. Yaghoobi).

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Nomenclature

$a,\ b,\ c,\ d$	constants	*Greek symbols*	
C_D	drag coefficient	$\alpha,\ \beta$	constants
D	particle diameter (m)	μ	dynamic viscosity (kg/m s)
g	acceleration due to gravity (m/s^2)	ρ	fluid density (kg/m^3)
m	particle mass (kg)	ρ_s	spherical particle density (kg/m^3)
Re	Reynolds number		
t	time (s)		
u	velocity (m/s)		

$$C_D = \alpha + \frac{\beta}{\mathrm{Re}}. \tag{1}$$

For specific values of α and β, this equation is valid in a wide range of Reynolds numbers, $0 \leqslant \mathrm{Re} \leqslant 10^5$. The analysis derived by the homotopy perturbation method (HPM) and Padé approximant. The solutions are compared with those derived by HPM [4] and the well-known fourth order Runge–Kutta method in order to verify the accuracy of the proposed method.

2. Problem statement

Consider a small, rigid, spherical, particle of diameter D, mass m and density ρ_s falling in an infinite extent of an incompressible Newtonian fluid of density ρ and viscosity μ. Let u represent the velocity of the particle at any instant time, t, and g the acceleration due to gravity. The unsteady motion of the particle in a fluid can be described by the Basset–Boussinesq–Ossen (BBO) equation. For a dense particle falling in light fluids and by assuming $\rho \ll \rho_s$, Basset History force is negligible. Thus, the equation of particle motion is given as

$$m\frac{du}{dt} = mg\left(1 - \frac{\rho}{\rho_s}\right) - \frac{1}{8}\pi D^2 \rho C_D u^2 - \frac{1}{12}\pi D^3 \rho \frac{du}{dt}, \tag{2}$$

where C_D is the drag coefficient. In the right hand side of Eq. (2), the first term represents the weight and buoyancy effects, the second term corresponds to drag resistance, and the last term is due to the added (virtual) mass effect which is due to acceleration of fluid around the particle.

The non-linear terms due to non-linearity nature of the drag coefficient, C_D is the main difficulty in solution of Eq. (2). Substituting Eq. (1) in Eq. (2) and by rearranging parameters, Eq. (2) could be rewritten as follow:

$$a\frac{du}{dt} + bu + cu^2 - d = 0, \quad u(0) = 0, \tag{3}$$

where

$$a = \left(m + \frac{1}{12}\pi D^3 \rho\right), \tag{4a}$$

$$b = \frac{\beta}{8}\pi D\mu, \tag{4b}$$

$$c = \frac{\alpha}{8}\pi D^2 \rho, \tag{4c}$$

$$d = mg\left(1 - \frac{\rho}{\rho_s}\right). \tag{4d}$$

3. Padé approximants

A Padé approximant is the ratio of two polynomials constructed from the coefficients of the Taylor series expansion of a function $u(x)$. The $[L/M]$ Padé approximants to a function $y(x)$ are given by [20]:

$$\left[\frac{L}{M}\right] = \frac{P_L(x)}{Q_M(x)}, \tag{5}$$

where $P_L(x)$ is polynomial of degree at most L and $Q_M(x)$ is a polynomial of degree at most M. The formal power series

$$y(x) = \sum_{i=1}^{\infty} a_i x^i, \tag{6}$$

$$y(x) - \frac{P_L(x)}{Q_M(x)} = O(x^{L+M+1}), \tag{7}$$

determine the coefficients of $P_L(x)$ and $Q_M(x)$ by the equation. Since we can clearly multiply the numerator and denominator by a constant and leave $[L/M]$ unchanged, we imposed the normalization condition

$$Q_M(0) = 1.0. \tag{8}$$

Finally, we require that $P_L(x)$ and $Q_M(x)$ have non-common factors. If we write the coefficient of $P_L(x)$ and $Q_M(x)$ as

$$P_L(x) = p_0 + p_1 x + p_2 x^2 + \cdots + p_L x^L, \tag{9a}$$

$$Q_M(x) = q_0 + q_1 x + q_2 x^2 + \cdots + q_M x^M, \tag{9b}$$

then by (10), we may multiply (7) by $Q_M(x)$, which linearizes the coefficient equations. We can write out (9) in more details as

$$\begin{aligned} &a_{L+1} + a_L q_1 + \cdots + a_{L-M+1} q_M = 0,\\ &a_{L+2} + a_{L+1} q_1 + \cdots + a_{L-M+2} q_M = 0,\\ &\vdots\\ &a_{L+M} + a_{L+M-1} q_1 + \cdots + a_L q_M = 0, \end{aligned} \tag{10a}$$

$$\begin{aligned} &a_0 = p_0,\\ &a_0 + a_0 q_1 = p_1,\\ &\vdots\\ &a_L + a_{L-1} q_1 + \cdots + a_0 q_L = p_L. \end{aligned} \tag{10b}$$

To solve these equations, we start with (10a), which is a set of linear equations for all the unknown q'. Once the q' are known, then (10b) gives and explicit formula for the unknown p', which complete the solution. If (10a) and (10b) are nonsingular, then we can solve them directly and obtain (11), where (11) holds, and if the lower index on a sum exceeds the upper, the sum is replaced by zero:

$$\left[\frac{L}{M}\right] = \frac{\det\begin{bmatrix} a_{L-M+1} & a_{L-M+2} & \cdots & a_{L+1}\\ \vdots & \vdots & \ddots & \vdots\\ a_L & a_{L+1} & \cdots & a_{L+M}\\ \sum_{j=M}^{L} a_{j-M} x^j & \sum_{j=M-1}^{L} a_{j-M+1} x^j & \cdots & \sum_{j=0}^{L} a_j x^j \end{bmatrix}}{\det\begin{bmatrix} a_{L-M+1} & a_{L-M+2} & \cdots & a_{L+1}\\ \vdots & \vdots & \ddots & \vdots\\ a_L & a_{L+1} & \cdots & a_{L+M}\\ x^M & x^{M-1} & \cdots & 1 \end{bmatrix}}. \tag{11}$$