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Original research paper

Vortex simulation for non-axisymmetric collision of a vortex ring with solid particles

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ABSTRACT

This study is concerned with the numerical simulation for the non-axisymmetric collision between a vortex ring and solid particles. The vortex ring convects with its self-induced velocity in a quiescent air, and the half part collides with spherical glass particles. The vortex method for gas-particle two-phase flow proposed by the authors in a prior paper is used for the simulation. The Reynolds number of the vortex ring is 2600, and the particle diameter is 50 μm. The Stokes number, defined as the ratio of the particle response time to the characteristic time of the vortex ring, is 0.74. The simulation clarifies that the particles induce the vortices, having an axis parallel to the convection direction of the vortex ring, inside the vortex ring and that pairs of the positive and negative vortex tubes appear. It also highlights that highly organized three-dimensional vortical structures composed of the streamwise vortices yield the rapid deformation and collapse of the vortex ring.

1. Introduction

Free turbulent gas flows loaded with solid particles are observed in various engineering devices, such as chemical reactors and solid rocket engines. The particle number density and velocity were measured in plane mixing layers [1,2], jets [3–5] and wakes [6,7]. The measurements made clear the gas turbulence modulation due to the particles as well as the relation between the large-scale eddies and the particle motion. A direct numerical simulation [8] and a large eddy simulation [9] for particle-laden jets were also performed to supplement the experimental work, and the effect of particle on the gas flow was explored.

It should be noted that vortex methods have been usefully applied to analyze single-phase flows. The methods can calculate directly the development of vortical structure, such as the formation and deformation of vortices, by tracing the motion of the vortex elements having vorticity through the Lagrangian approach. Therefore, they have been favorably applied to free turbulent flows, in which the organized large-scale eddies play a dominant role. One of the authors [10] proposed a vortex method for gas-particle two-phase flow. The method was applied to jets [11,12] and a plane mixing layer [13] to simulate the relation between the large-scale eddies and the particle motion as well as the change in the gas flow due to the particle. The vortex method was also successfully employed to compute the particulate jet induced by the particles falling in a quiescent air [14].

In a jet issuing from a round nozzle, vortex rings exist at regular intervals in the streamwise direction on the jet periphery near the nozzle. The vortex rings deform with increasing the streamwise distance, and they collapse eventually at the end of the core region. Such behavior of the vortex ring affects the jet characteristics. It is gradually being clarified that the vortices, having an axis parallel to the streamwise direction, are deeply concerned with the deformation and collapse of the vortex ring [15]. The development of gas-particle two-phase jet seems to be affected by the vortex rings. But the particle motion near a vortex ring and the effect of the particle on the vortex ring have been remained unclarified.

In a prior paper [16], the authors simulated the collision of a vortex ring, convecting with its self-induced velocity in a quiescent air, with small glass particles by their vortex method. The whole vortex ring collides with the particles arranged uniformly in planes perpendicular to the convection direction of the vortex ring. The simulation made clear the deformation and collapse of the vortex ring due to the particles. But it yields less knowledge on the collision between a part of vortex ring and solid particles, which seems to occur frequently in particle-laden jets issuing from a round nozzle.

This study explores the non-axisymmetric collision between a vortex ring and solid particles by the vortex method. The vortex ring convects with its self-induced velocity in a quiescent air, and the half part collides with spherical glass particles. The

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Nomenclature

d	particle diameter
D	diameter of vortex core
$\boldsymbol{f}_D$	drag force acting on particle
$\boldsymbol{F}_D$	force exerted by particle acting on air per unit volume
$\boldsymbol{g}$	gravitational acceleration
p	pressure
t	time
t^*	non-dimensional time = tU_c/D_0
$\boldsymbol{u}$	velocity
U_c	convective velocity of vortex ring when colliding with particles
x, y, z	orthogonal coordinates
γ	strength of vortex element
Δt	time increment $= 0.01D_0/U_c$
ν	kinematic viscosity
ρ	density
σ	core radius of vortex element
τ	characteristic time
ω	vorticity = $\nabla \times \boldsymbol{u}_g$

Subscripts

0	initial value
g	gas
p	particle
x, y, z	component in direction of x, y or z

Reynolds number of the vortex ring is 2600, and the particle diameter is 50 μm. The Stokes number, defined as the ratio of the particle response time to the characteristic time of the vortex ring, is 0.74. The simulation clarifies that the particles induce the vortices, having an axis parallel to the convection direction of the vortex ring, inside the vortex ring and that pairs of the positive and negative vortex tubes appear. It also highlights that highly organized three-dimensional vortical structures composed of the streamwise vortices yield the rapid deformation and collapse of the vortex ring.

2. Basic equations

2.1. Assumptions

The following assumptions are employed for the simulation.

(1) The gas is incompressible.
(2) The particle density is much larger than the gas.
(3) The particle has a spherical shape with uniform diameter and density.
(4) The collision between the particles is negligible.

2.2. Governing equations for gas and particle

The conservation equations for the mass and momentum of the gas are expressed as follows under assumption (1):

$$\nabla \cdot \boldsymbol{u}_g = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}_g}{\partial t} + (\boldsymbol{u}_g \cdot \nabla)\boldsymbol{u}_g = -\frac{1}{\rho_g}\nabla p + \nu \nabla^2 \boldsymbol{u}_g - \frac{1}{\rho_g}\boldsymbol{F}_D \tag{2}$$

where $\boldsymbol{F}_D$ is the force exerted by the particle acting on the gas per unit volume.

Using assumption (2), the dominant forces on the particle are the drag and gravitational forces, while the virtual mass force, the Basset force and the pressure gradient force are negligible [17]. The lift force is neglected with the reference to the studies simulating the particle motion in a jet [18], a plane wake [6], and mixing layers [1,17]. Consequently, the equation of motion for a particle (mass m) is written as

$$m\frac{d\boldsymbol{u}_p}{dt} = \boldsymbol{f}_D + m\boldsymbol{g} \tag{3}$$

where the drag force $\boldsymbol{f}_D$ is given by the following from assumption (3):

$$\boldsymbol{f}_D = (\pi d^2 \rho_g / 8) C_D |\boldsymbol{u}_g - \boldsymbol{u}_p| (\boldsymbol{u}_g - \boldsymbol{u}_p) \tag{4}$$

Here, d is the particle diameter, and the drag coefficient C_D is estimated as [19]

$$C_D = (24/\mathrm{Re}_p)(1 + 0.15\mathrm{Re}_p^{0.687}) \tag{5}$$

where $\mathrm{Re}_p = d\,|\boldsymbol{u}_g - \boldsymbol{u}_p|/\nu$.

For the simultaneous calculation of Eqs. (1)–(3), a vortex method is used to solve Eqs. (1) and (2), and the Lagrangian method is applied to Eq. (3).

2.3. Discretization of vorticity field by vortex element

When taking the curl of Eq. (2) and substituting Eq. (1) into the resultant equation, the vorticity equation for the gas is derived:

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\boldsymbol{u}_g + \nu \nabla^2 \boldsymbol{\omega} - \frac{1}{\rho_g}\nabla \times \boldsymbol{F}_D \tag{6}$$

where $\boldsymbol{\omega}$ is the vorticity.

The gas velocity $\boldsymbol{u}_g$ at $\boldsymbol{x}$ is given by the Biot–Savart equation.

$$\boldsymbol{u}_g(\boldsymbol{x}) = -\frac{1}{4\pi}\int \frac{(\boldsymbol{x} - \boldsymbol{x}') \times \boldsymbol{\omega}(\boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|^3} dV(\boldsymbol{x}') \tag{7}$$

The gas vorticity field is discretized by vortex elements. The vortex element has a cylindrical shape, while the vorticity distribution is spherical with a finite core radius.

When the vortex element α at $\boldsymbol{x}^\alpha$ is supposed to have the core radius σ_α, the vorticity at $\boldsymbol{x}$ induced by the element is expressed as

$$\boldsymbol{\omega}^\alpha(\boldsymbol{x}) = \frac{\boldsymbol{\gamma}^\alpha}{\sigma_\alpha^3} f\left(\frac{|\boldsymbol{x} - \boldsymbol{x}^\alpha|}{\sigma_\alpha}\right) \tag{8}$$

where $\boldsymbol{\gamma}^\alpha$ is the strength of vortex element. The core distribution function $f(\varepsilon)$ is given by the following equation proposed for single-phase flow [20]:

$$f(\varepsilon) = \frac{15}{8\pi(\varepsilon^2 + 1)^{7/2}} \tag{9}$$

2.4. Convection of vortex element and evolution of vortex strength

When the gas vorticity field is discretized into a set of N_v vortex elements, the gas velocity $\boldsymbol{u}_g(\boldsymbol{x})$ is given by the following equation derived from Eqs. (7) and (8)

$$\boldsymbol{u}_g(\boldsymbol{x}) = -\frac{1}{4\pi}\sum_{\alpha=1}^{N_v} \frac{(\boldsymbol{x} - \boldsymbol{x}^\alpha) \times \boldsymbol{\gamma}^\alpha}{|\boldsymbol{x} - \boldsymbol{x}^\alpha|^3} g\left(\frac{|\boldsymbol{x} - \boldsymbol{x}^\alpha|}{\sigma_\alpha}\right) \tag{10}$$

where the function $g(\varepsilon)$ is determined as