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Colossal anelasticity in polycrystals deforming under conditions of diffusional creep

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Abstract

Since diffusional creep viscosity depends strongly on the grain size, a variability in the grain size in a polycrystal can generate significant internal stresses. Upon unloading, the non-uniform internal stresses recover, leading to anelastic strains. The anelastic strains can be very large, approaching 100 times the elastic strains. We solve the case of bimodal viscosity in closed form to highlight the features of this type of anelasticity. The results are compared to the anelastic behavior of two-phase superplastic alloys. In such alloys the spatial variability in the diffusional viscosity can arise from the variability in the activation energy of grain boundary diffusion, which, because of its Arrhenius nature, can produce a much higher degree of variability than the grain size.

Keywords: Anelasticity; Diffusional creep; Grain size distribution

1. Introduction

The time-dependent recovery of strain, generally known as viscoelasticity [1], is called anelastic deformation in the metals literature [2]. This special nomenclature has its origin in the specific atomistic and microstructural mechanisms of anelasticity; the mechanistic modeling then unfolds into a prediction for the magnitude and the time constant for strain recovery. Examples include the movement of carbon atoms between tetrahedral and octahedral sites in body-centered-cubic iron [3], the to-and-fro bowing of dislocations that are pinned [4], and sliding at grain boundaries in polycrystals that is accommodated by elastic deformation in the neighboring grains [5]. These mechanisms follow the Voigt representation as shown on the left in Fig. 1a. Here the dashpot, η_{VG}, represents the viscosity for the accumulation of anelastic strain, and E_{VG} is the anelastic modulus. The spring E_o refers to “pure” elastic deformation, which is time independent.

The right-hand side in Fig. 1 illustrates a different type of anelastic behavior. Here a series of dashpots are connected in parallel, each with a different viscosity. In steady-state deformation, each dashpot deforms at the same rate but, because of its own unique viscosity, supports a load that is different from the others. These loads also produce elastic deformation which is carried by the springs that are attached in series to the dashpots. At the point of unloading, each spring will support a different stress which will cause a recovery of strain. The recovery will be time-dependent since it will be controlled by the viscosity of the dashpots. In this paper we analyze this viscoelastic problem using linear descriptions of the viscosity and elastic constants, as can be assumed for a polycrystal deforming by diffusional creep. In Coble creep, the diffusional creep viscosity is proportional to the third power of the grain size. Therefore, a spatial variability in the grain size can give rise to the series of spring–dashpot elements represents shown in Fig. 1b. This Maxwell model for anelasticity assumes that the polycrystal can be divided into columns of regions each with its own grain size, as if in a one-dimensional composite. In spite of this simplification we are able

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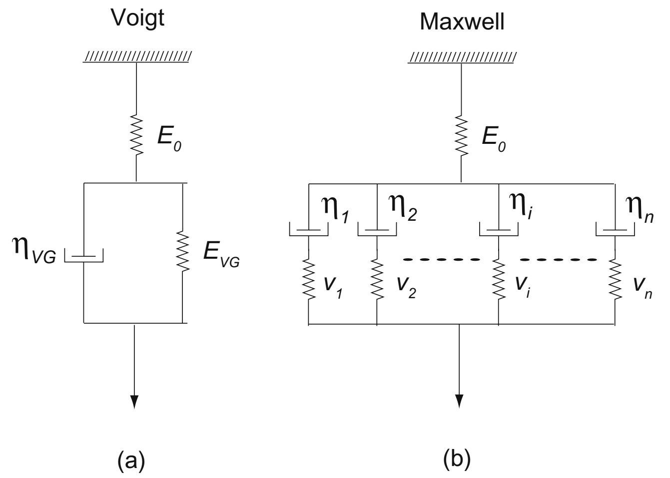

Fig. 1. Voigt and Maxwell models for anelastic deformation. This paper considers the Maxwell model.

to obtain meaningful insights and results that can be compared with experiment.

It is to be noted that the variability in diffusional viscosity can also arise from different diffusion coefficients that operate in a multiphase polycrystal. A two-phase material, for example, contains at least three kinds of boundaries, two between the like phases (AA and BB), and one between the unlike phases (AB). Indeed the AB type boundaries may have a very large variability among them since the segregation of the chemical species is likely to vary from one boundary structure to another, due to different heats of segregation. These differences will affect the local activation energy for diffusion, and since the diffusivity depends exponentially on the activation energy, even a small variability in the activation energy can lead to a spread in viscosities that is far wider than those arising from the grain size. (It is interesting to note that very large anelastic strains, as described below, have been measured in two-phase superplastic alloys.)

The most prominent studies of large anelastic strains have been reported in two-phase superplastic alloys. Schneibel and Hazzledine [6] measured anelastic recovery of 1% strain in a Sn–Pb alloy, equal to 130 times the elastic strain. Later, Vale [7] measured an anelastic strain of 0.5% in a eutectoid Zn–Al alloy. Todd [8] has measured anelastic strain up to 290 times the elastic strain in the same alloy, which was separated into two regimes, the first a short-term relaxation which yields an anelastic strain which is similar to the measurements of Schneibel and Hazzledine [6], and then a longer-term relaxation which accounts for the remainder of the anelasticity. The stress dependence of these two regimes is also different. The short-term strain is linear in stress, meaning that the anelastic strain, when expressed as a multiple of the elastic strain, remains constant. The long-term anelastic strain, however, approached a constant absolute value, being relatively independent of the applied stress. The relaxation times for the short-term and long-term strain recovery were also different, being proportional to the square of the grain size for the short-term strain, and the fourth power of the grain size for the long-term strain.

These long relaxations and the large anelastic strains have been explained by the opening and sintering of cavities [7]. However, such an interpretation was discounted by Todd and Hazzledine [9] since the anelasticity was present in both tension and compression experiments, and since the Poisson's ratio for the anelasticity was equal to 0.5, which represents pure shear deformation at a constant volume.

It is also noteworthy that the rate of increase of the absolute values of the anelastic strains became weaker at higher applied stresses [8]. The present analysis can account for this effect since at higher stress the larger grains, which would bear a high load, would transition into plastic deformation or power-law creep [10], thereby reducing the variability in the redistribution of the applied stress.

The analysis in this paper considers a bimodal distribution of the viscosity. This special case, though approximate, can be solved in closed form. The closed-form solution permits the derivation of limiting cases, when the viscosities and the volume fractions of the two domains are very different. Note that the variability in the viscosity can arise from both the grain size and the diffusivity. The latter is likely to be more important in the case of two-phase alloys.

The present approach also permits the study of time, temperature and grain size effects in a unified way, since they are all related to one another via the viscosity. Indeed, the analysis can extend from nanograin materials [11], to very large grain structures, such as of olivine in the upper mantle [12,13], and from short-term laboratory timescales to very long timescales, that may apply to geological measurements.

2. Nomenclature

We consider the general case for the Maxwell model, shown Fig. 1b. The polycrystal is assumed to consist of *n* elements, such that *i* = 1 to *n*, which leads to the following nomenclature for the parameters for each of the elements:

ε_i	total tensile strain of the *i*th partition with two components $\varepsilon_{i,1}$ *and* ε_2
$\varepsilon_{i,1}$	is the strain from elastic deformation
$\varepsilon_{i,2}$	is the strain from the viscous deformation
σ_i	tensile stress distributed to the *i*th partition
η_i	viscosity of the *i*th partition
v_i	the volume fraction of the *i*th partition, such that $\sum_{i=1,n} v_i = 1$

The global parameters in the problem are as follows:

E_0	Young's modulus of the material
σ_∞	applied tensile stress
Δ_A	the ratio of the anelastic strain to the elastic strain