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Stress-induced destabilization of solidification and melting fronts

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Abstract

The morphological evolution of the initially planar solidification and melting fronts of a thin liquid film in a stressed binary alloy has been investigated when diffusion only proceeds in the liquid phase. A linear stability analysis has been performed and the diffusion-controlled evolution of the shape of both fronts has been characterized. The destabilizing effect of stress on the profiles of the interfaces has been identified for a liquid film at rest when the solid is submitted to constant stress and when it is migrating, due to stress gradient, in the hypothesis where concentration field of solute satisfy Laplace's equation. The possibility of roughness formation in the early beginning of the development of the solid–liquid interfaces has been finally discussed for alloys in the context of a liquid film migration mechanism.

Keywords: Diffusion in liquids; Interface migration; Continuum mechanics; Morphological evolution; Analytic methods

1. Introduction

Liquid film migration (LFM) has been widely studied because of its intensive use in a number of industrial processes of great practical importance. It has been observed, for example, in solid alloys during sintering when a liquid phase is present [1–3], during partial melting of Cu–In solids [4] or isothermal annealing of Al–Cu alloys after quenching [5]. The migration of a liquid film takes place when both solidification and melting fronts are moving in the same direction due to solute diffusion through the liquid phase, the interface velocity being typically for the considered solids of the order of 10^{-8}–10^{-7} m s^{-1}. It has been demonstrated that in W–Ni or Mo–Ni systems, for example, the difference between equilibrium states at both fronts which is at the origin of the driving force for LFM, may result from coherency stress appearing ahead of the melting front due to a sharp concentration profile [6]. In the hypothesis where the temperature and chemical compositions are kept constant at the interfaces, an exact solution describing the steady-state motion of two co-focal parabolic fronts has been theoretically determined by Temkin [7]. Considering capillarity effects, a selection theory for the velocity of these parabolic fronts has been then developed by Brener and Temkin [8] which includes anisotropic surface tension features. The shape evolution of a solid–liquid planar interface during unidirectional solidification has been studied by Mullins and Serkerka [9]. It has been found by these authors that the interface may undergo the so-called morphological Mullins–Serkerka (MS) instability. An interface at rest may also be unstable with respect to shape perturbation when the solid is uniaxially stressed, undergoing in this case the Asaro–Tiller–Grinfeld (ATG) instability [10,11]. The coupling between both MS and ATG instabilities has been investigated and a weak stress (of the order of 10^{-3} bar) has been found to deeply modify the MS instability [12,13].

In this paper, the morphological evolution of two initially planar solid–liquid interfaces of a migrating thin liquid film in a stressed binary alloy has been investigated when diffusion occurs in the liquid phase. Assuming that the interface energies are isotropic, the conditions for which the liquid film migrates at constant width have been first examined. The destabilizing effect of constant stress and stress gradient on both solid–liquid interface profiles has been then analyzed in their linear regime of evolution.

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The possibility of application of these results to alloys where, during LFM, the coherency stress is concentrated in a thin layer ahead of the melting front is finally discussed.

2. Modelling

A liquid film (L) of initial thickness h is considered in a binary dilute alloy made of two substitutional components and labelled S_1 for $-\infty \leqslant y \leqslant y_1$ and S_2 for $+\infty \geqslant y \geqslant y_2$, where y_1 and y_2 are the coordinates along the $(0y)$ axis of the solidification and melting fronts, respectively (see Fig. 1 for axes). The changes of density during solidification and melting as well as the flow in the liquid are ignored. The temperature in both phases is assumed to be constant and linear elasticity theory is used to describe the strain field in the alloy. The interfacial stress at both solid–liquid interfaces and the gravity effects are neglected, the initial reference stress being then defined as follows: $|P_{S_1}| = |P_{S_2}| = |P_L| = 0$, where P_{S_i} and P_L are the initial pressures in S_i and L, respectively, with $i = 1, 2$. A constant stress $T^i_{xx} = T^i_0$ with $(|T^i_0| >> |P_L|)$ is applied to each part of the solid phase S_i, the Young's modulus E and Poisson's ratio ν of the isotropic solid alloy being the same in S_1 and S_2. The vacancies are neglected and the concentration of solute during the diffusion-controlled isothermal solidification and melting processes is labelled C_{S_i} in the solid and C_L in the liquid. The solute diffusion in the solid phase has not been considered and the solute concentrations at the interfaces in the solid phase are set to given constant values that may not be the equilibrium ones [16]. In the coordinate system $(x, \tilde{y} = y - Vt, z)$ attached to the S_1/L interface, the concentration of solute in the liquid phase is assumed to satisfy the steady-state equation of diffusion [9]:

$$\Delta C_L + \frac{2}{l_L}\frac{\partial C_L}{\partial \tilde{y}} = 0, \tag{1}$$

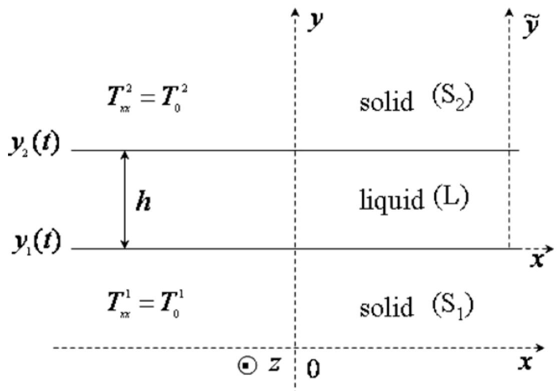

Fig. 1. A thin liquid film of thickness h in a solid alloy. Each part S_i of the solid is submitted to a constant stress $T^i_{xx} = T^i_0$. The fixed and moving frames attached to the S_1/L interface are labelled (x, y, z) and $(x, \tilde{y}, z)$, respectively.

where the length l_L is defined as $l_L = 2D_L/V$, with V the velocity of the migrating film and D_L the diffusion coefficient in the liquid phase. Assuming also that the local thermodynamic equilibrium holds at both fronts, the Gibbs–Thomson equation gives the following boundary conditions at each solid–liquid interface [8,9,12,15,16]:

$$\delta C_L|_{int} = C_L|_{int} - C^0_L = \Gamma_L \mathscr{G}^i, \tag{2}$$

where C^0_L and C^0_S are the equilibrium concentrations of the solute in the liquid and solid phases at a planar stress-free interface, respectively, $\Gamma_L = \gamma T_M/(Lm)$, with T_M the melting temperature of the pure solvent, γ the constant interface energy per unit surface, L the latent heat per volume. The liquidus slope m is assumed to be negative in the case where the solubility of solute in the solid is less than the one in the liquid, i.e. for $C^0_L \geqslant C^0_S$. The $\mathscr{G}^i$ function is defined by:

$$\mathscr{G}^i = \kappa + \frac{1}{2\gamma} S_{klmn} T^i_{kl} T^i_{mn}, \tag{3}$$

with κ the interface curvature assumed to be positive for a convex profile and S_{klmn} the components of the stiffness tensor of the solid, the $k, l, \underline{m}, n$ repeated indices being summed over. The stress tensor $(\overline{\overline{T}}_i)$ in S_i whose components are T^i_{kl} has been determined in the plane-strain hypothesis assuming that the mechanical equilibrium condition,

$$\overline{\overline{T}}_i \cdot \mathbf{n}^i = 0, \tag{4}$$

holds at any time on both interfaces, with $\mathbf{n}^i$ the normal to the ith interface pointing into the liquid. Considering a biharmonic function φ^i such that [17]:

$$\Delta\Delta\varphi^i = 0, \tag{5}$$

the stress components in S_i are given by:

$$T^i_{xx} = \frac{\partial^2 \varphi^i}{\partial \tilde{y}^2}, \quad T^i_{\tilde{y}\tilde{y}} = \frac{\partial^2 \varphi^i}{\partial x^2}, \quad T^i_{x\tilde{y}} = -\frac{\partial^2 \varphi^i}{\partial x \partial \tilde{y}}. \tag{6}$$

Finally, the normal velocity v^i_n of the ith interface is given by [9,14]:

$$(C_{S_i} - C_L)|_{int} v^i_{n^i} = D_L \nabla C_L|_{int} \cdot \mathbf{n}^i, \tag{7}$$

where the concentrations and their gradient are evaluated at the ith interface. The condition on T^i_0 stresses for which both planar interfaces are migrating at the same velocity such that the film thickness h can be taken as a constant have been first determined in the steady-state regime. Solving Eq. (1) in the case where the shape of both planar interfaces are time independent, the expressions of the concentration field of solute in the liquid are given by:

$$C^{(0)}_L(\tilde{y}) = A^{(0)}_L e^{-2\tilde{y}/l_L} + B^{(0)}_L. \tag{8}$$

Using Eq. (2), the coefficients $A^{(0)}_L$ and $B^{(0)}_L$ have been determined considering a strain field in the solid characterized by the potential function satisfying Eq. (5):

$$\varphi^{i,(0)}(\tilde{y}) = \frac{T^i_0}{2}\tilde{y}^2. \tag{9}$$