Advanced Powder Technology 24 (2013) 306–310

Contents lists available at SciVerse ScienceDirect

Advanced Powder Technology

journal homepage: www.elsevier.com/locate/apt

Original Research Paper

Dynamic shape factor for particles of various shapes in the intermediate settling regime

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ARTICLE INFO

Article history:
Received 4 June 2012
Received in revised form 30 July 2012
Accepted 2 August 2012
Available online 23 September 2012

Keywords:
Dynamic shape factor
Intermediate settling regime
Terminal settling velocity
Wall effect
Elongated particles

ABSTRACT

There are a number of shape factors available in the literature to describe the deviation of non-spherical particles to spheres. Dynamic shape factor (χ) is the most suitable parameter for flow applications as it takes into account the dynamic properties of the non-spherical particles. In this study, dynamic shape factors for spheres, cubes, cuboids and cylinders settling in the transition regime ($2 < Re_t < 500$) are determined experimentally. Wall effects on the terminal settling velocity are accounted based on literature models. χ for spheres and cubes are found to be constant with particle size when conventional wall effect models are followed. χ for cuboids and cylinders, on the other hand, are found to be a function of the particle characteristic length-to-column diameter ratio (l/D) and the particle aspect ratio (l/d). An empirical correlation is developed to estimate χ for both cuboids and cylinders settling in the intermediate regime. The correlation can provide a low average error of 3% for all the cuboids and cylinders used in this study.

1. Introduction

Shape factors have been developed over the years to relate the non-spherical property to that of sphere. A summary of various shape factors reported in the literature is shown in Table 1. The simplest and most widely used shape factor is the sphericity (ψ), defined as the ratio of the surface area of an equivalent volume sphere to that of the particle. Based on similar concept, circularity (defined as the ratio of the perimeter of a projected area equivalent circle to that of the particle projected area) and not-roundness (defined as the variance of the deviation of the particle radius from either a circle in two dimensions or a sphere in three dimensions) are also developed. However, most applications of non-spherical particles are under flow conditions and these shape factors either do not consider or require a presumed particle settling orientation. It is difficult to use these shape factors to represent the actual dynamic properties of the non-spherical particles. Dynamic shape factor (χ), on the other hand, is defined based on the ratio of the drag force experienced by a non-spherical particle to the drag force experienced by a volume-equivalent sphere travelling at the same velocity and medium [1]. It is the most suitable parameter for flow applications. The drag force on a sphere can be described by three expressions depending on the settling regime. In the literature, dynamic shape factor is further sub-classified as Stokes shape factor [2–4] and Newton's shape factor [5]. Dynamic shape factor can be used to develop models to predict the drag force and to design particles with the desired dynamic properties [6]. However, there is a lack of studies determining the dynamic shape factor in the transition regime. This information will be particularly useful for drug inhalation applications that involve carrier particles whose settling will be under the intermediate regime [6–11]. Therefore, this study aims to determine the dynamic shape factors for spheres, cubes, cuboids and cylinders settling in the intermediate regime ($2 < Re_t < 500$) and to develop appropriate models to unify the dynamic shape factors for cuboids and cylinders having various aspect ratios.

2. Materials and methods

Based on the definition of χ, a general equation can be written as:

$$\chi = \frac{V(\rho_p - \rho_f)g}{\pi/8 C_D \rho_f u_t^2 d_e^2} \quad (1)$$

where the numerator is the drag force of the non-spherical particle obtained by balancing the gravitational and buoyancy forces and the denominator is the drag force of a volume-equivalent sphere. C_D in the intermediate regime can be expressed as [18]:

$$C_D = \frac{18.5}{Re_t^{0.6}} \quad (2)$$

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0921-8831/$ - see front matter
http://dx.doi.org/10.1016/j.apt.2012.08.001

Nomenclature

C_1	correlation constant used in Eq. (4)
C_2	correlation constant used in Eq. (4)
C_3	correlation constant used in Eq. (4)
C_D	drag coefficient
d	minor dimension of particle
d_A	diameter of surface-equivalent sphere
d_e	diameter of volume-equivalent sphere
D	column diameter
g	gravitational constant
l	major dimension of particle
P_p	projected perimeter of particle along the axis of settling
Re_t	particle Reynolds number
$Re_{t\infty}$	particle Reynolds number at unbounded fluid medium
u_t	particle terminal velocity
$u_{t\infty}$	unbounded particle terminal velocity
V	volume of particle
λ	diameter of sphere divided by internal diameter of cylindrical tube
ρ_p	density of particle
ρ_f	density of fluid medium
μ	viscosity of fluid medium
χ	dynamic shape factor
χ_0	correlation constant used in Eqs. (4) and (5)
ψ	sphericity

where Re_t is defined as $\frac{\rho_f u_t d_e}{\mu}$. Substituting Eq. (2) into Eq. (1) and expressing V and Re_t in their basic form, χ in the intermediate regime can then be obtained by the equation:

$$\chi = 0.072\frac{d_e^{1.6}(\rho_p - \rho_f)g}{\rho_f^{0.4}\mu^{0.6}u_t^{1.4}} \quad (3)$$

Therefore, χ can be determined from the measurement of the particle terminal settling velocity.

Four particle shapes, namely sphere, cube, cuboid and cylinder are considered in this study. Spheres, cubes and cuboids are molded using polymer clay (Sculpey III, USA) while cylinders are obtained by cutting up rubber rods (Mitsubishi Pencil, Japan). A list of the dimensions and the particle properties of all the particles used in the study is shown in Table 2.

The experimental setup for particle terminal settling velocity measurement is shown in Fig. 1. Particle terminal settling velocity is measured in an acrylic column 1 m in height and 70 mm in diameter. MobilTherm 605 (Mobil, Singapore) of density 870.0 kg/m^3 and viscosity 0.0235 kg/m s is used as the liquid medium to allow particle settling in the intermediate regime ($2 < \mathrm{Re}_t < 500$). Particles are introduced 3 cm below the liquid surface to minimize the disturbance of the liquid medium. A high-speed camera (Olympus, I-speed) is used to capture the particle settling motion at a frame rate of 1000 Hz. A 500 W spotlight is used as the light source. High-speed video is captured at the bottom of the column to ensure the achievement of terminal settling velocity. While experimental results have shown that all the particles can achieve the terminal settling velocity within a height of 200 mm, an average terminal settling velocity is measured between 700 mm and 800 mm from the liquid surface. It is to minimize the fluctuation in settling velocity due to constant reorientation of particles. Minimum of 8 measurements are performed for each particle to ensure repeatability of the measured results.

Since the experimental determination of the particle terminal settling velocity is performed in a finite column, the presence of the finite wall can cause a retarding effect on the settling of the particle. In particular, as the characteristic length-to-column diameter ratio (l/D) for cuboids and cylinders increases, the particle will settle at an orientation different from the natural settling orientation. Studies have shown that particles settling in $5.5 < \mathrm{Re}_{t\infty} < 200$ tend to have a stable settling orientations correspond to the maximum drag [12,19]. Thus, a correction of the wall effect on the particle terminal settling velocity is necessary. Chhabra et al. [20] has found a large average error of 12–27% by comparing various wall effect models with open literature data over a wide range of λ. A comparison of the sphere settling results obtained in the current study with three literature wall effect models developed for the intermediate regime [21–23] indicates that the Di Felice and Kehlenbeck [23] model delivers the lowest error. Therefore, the Di Felice and Kehlenbeck model is also used for cubes. For cuboids and cylinders, due to a lack of available models, the one developed in Lau et al. [24], that gives the best match to the current experimental conditions, will be used.

3. Results and discussions

All the u_t obtained experimentally are corrected for the wall effect. The unbounded particle Reynolds number ranges between $50 < \mathrm{Re}_{t\infty} < 167$, which is within the intermediate regime. Thus, it is justifiable to determine χ of all the particles utilized in the study using Eq. (3). The experimentally determined χ are plotted in Fig. 2. It can be seen that χ of spheres and cubes are fairly constant with different d_e. It shows that the settling orientation of spheres and cubes is independent of wall effect and the Di Felice and Kehlenbeck model is capable of describing the settling properties of spheres and cubes practically in the presence of wall effect. The average χ of spheres is found to be 0.98 and it agrees reasonably well with the theoretical value of 1. It can demonstrate that the treatment of the experimental data can represent the unbounded

Table 1
A list of commonly used shape factors.

Shape factor	Definition
Sphericity (ψ)	Defined as the ratio of the surface area of an equivalent volume sphere to that of the particle [12–14]
Not-roundness	Defined as the variance of the deviation of the particle radius from either a circle in two dimensions or a sphere in three dimensions [15]
Circularity	Defined as $\frac{\pi d_A}{P_p}$, where d_A is the diameter of the surface-equivalent sphere and P_p is the projected perimeter of particle along the axis of settling [16]
Dynamic shape factor (χ)	Defined as the ratio of C_D of non-spherical particle to C_D of sphere [5,13,14,17]