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Original Paper

Numerical studies of particle segregation in a rotating drum based on Eulerian continuum approach

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ABSTRACT

Solid–solid–gas three-phase particle segregation in a half-filled rotating drum is simulated using Eulerian continuum approach coupling the kinetic theory of granular flow. A dynamic angle of repose fitting (DARF) method is proposed to determine granular kinetic viscosities of particles of six different sizes moving in the drum rotating at 10 rpm, 20 rpm or 30 rpm. The DARF granular kinetic viscosity increases and decreases with the increasing of particle size and drum rotational speed, respectively. The determined DARF granular viscosity values are used to simulate size-induced particle segregation in a rotating drum. The simulated small-particle-rich segregation structure shows a central small-particle-rich band together with two small-particle-rich side wings. The size of the wings decreases with the increasing of the drum rotational speed. The formation of radial segregation core and axial segregation bands qualitatively agree with the experimental observations.

1. Introduction

The rotating drums are commonly used in industries for drying, mixing or granulation [11]. Granular flow regime in a rotating drum includes, at least, slipping, avalanching, rolling, cascading, cataracting and centrifuging [24]. Experimental and numerical methods have been proposed to study granular flows in rotating drums. Non-invasive experimental techniques include, for example, Magnetic Resonance Imaging [18] and Positron Emission Particle Tracking [23]. However, these experimental methods are limited in the size of observation, and applicable only for radioactive powders, not readily for ordinary granular flow studies. The rapid development in computational powers allows computer simulations to provide as an effective tool to study granular flows. Discrete Element Method (DEM) and Eulerian Continuum Approach (ECA) are two of the most popular methods for the numerical studies of granular flows. In DEM simulations, the forces acting on every particle in the system are calculated based on certain particle–particle interaction laws [2]. In ECA simulations, particles are treated as continuous fluid and granular flows are modelled as continuous fluid flows [27]. Although DEM simulations provide microscopic information at particle level, ECA simulations require less computational time and are preferred for large scale granular flow modelling.

Particle mixing in a rotating drum has been extensively studied. Granular mixtures of different sizes, densities, roughness or elasticity have tendency to segregate in a rotating drum [28]. When binary granular mixture of two different sizes is mixed in a rotating drum, small particles are quickly collected at the centre of the bed, forming a segregation core due to the percolation mechanism. With further revolutions, large particles are likely to stay in the region immediately next to the two end walls and alternative small-particle-rich bands and large-particle-rich bands are formed at the bed surface [21,13].

Numerical studies of particle segregation in a rotating drum using ECA are conducted in this study. The granular interactions are modelled based on the kinetic theory of granular flow. However, ECA simulation coupling the kinetic theory of granular flow is generally only applicable for dilute granular flow studies [6]. Most modelling work on dense granular flows in rotating drum uses DEM simulations, for example, Yamane et al. [29] and Arntz et al. [1].

Although there are limited works using ECA coupling the kinetic theory of granular flow to model granular flows in a rotating drum [12,4,7], there are still questions on the suitability of using the kinetic theory of granular flow to model dense granular flows in rotating drums. The local constitutive law developed by French group GDR MiDi [9] has been used to model dense shearing granular flows by Jop et al. [17]. In their model, the friction coefficient depends on a single dimensionless inertial parameter *I*, which can be interpreted as the ratio between the time scale given by the shear rate and the time scale related to the confining pressure. The local rheology constitutive equation of GDR MiDi was incorporated into the kinetic theory of granular flow by Jenkins and Berzi

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Nomenclature

C_D	drag coefficient (–)	μ	viscosity (kg m^{-1} s^{-1})
$C_{fr,sisj}$	friction coefficient between *i*th and *j*th solids-phase particles (–)	$\mu_{s,g}$	granular viscosity (kg m^{-1} s^{-1})
d_s	particle diameter (m)	ρ	density (kg m^{-3})
e_{ss}	restitution coefficient (–)	Φ	inter-phase energy exchange (kg s^{-3} m^{-1})
$g_{0,ss}$	radial distribution function (–)	$\vec{I}$	unit vector (–)
I_{2D}	second invariant of the deviatoric stress tensor (–)	$\vec{g}$	gravitational acceleration (m s^{-2})
K	inter-phase exchange coefficient (–)	$\vec{v}$	velocity (m s^{-1})
$k_{\theta s}$	diffusion coefficient (–)	$\bar{\bar{\tau}}$	stress–strain tensors (Pa)
P	pressure (Pa)		
Re_s	relative Reynolds number (–)	*Indices*	
α	volume fraction (–)	g	gas phase (–)
β	angle of internal friction (°)	si	*i*th component of solids phase (–)
$\gamma_{\theta s}$	rate of collisional energy dissipation (kg s^{-3} m^{-1})	sj	*j*th component of solids phase (–)
θ_s	granular temperature (m^2 s^{-2})		
λ	bulk viscosity (kg m^{-1} s^{-1})		

[15] and Berzi and Jenkins [3] to model dense granular flow down an inclined plane by assuming an algebraic balance between production and dissipation of fluctuation energy holds everywhere. However, the work of Félix et al. [8] concluded that granular flows in rotating drums are sensitive to the ratio between the drum and particle diameters and it is not possible to deduce a local particle rheology from experiments in a rotating drum. Thus, the local constitutive law developed by GDR MiDi group was not suitable to be used to simulate granular flows in rotating drums in the current work. In this study, the value of the kinetic viscosity of granular flows in the kinetic theory of granular flow model for the dense granular flows in the rotating drum is critically discussed. ECA simulations coupling the kinetic theory of granular flow are used to simulate particle segregation in a rotating drum.

2. Mathematical model

2.1. ECA model coupling the kinetic theory of granular flow

ECA coupling the kinetic theory of granular flow is used to simulate the motion of two granular materials in a rotating drum. Since the interstitial fluid in rotating drums is not always air and its influences on granular flow behaviour can be important [20], the gas phase is also considered in this work in order to show the importance of the interstitial fluid in granular flow. Thus, three continuum phases are considered in the entire domain: solids phase 1, solids phase 2 and gas phase. The equations of continuity for the gas and the solids phases are

$$\frac{\partial}{\partial t}\cdot(\alpha_g\rho_g) + \nabla\cdot(\alpha_g\rho_g\vec{v}_g) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\cdot(\alpha_{si}\rho_{si}) + \nabla\cdot(\alpha_{si}\rho_{si}\vec{v}_{si}) = 0 \tag{2}$$

respectively; where ρ, α and $\vec{v}$ are the density, the volume fraction and the velocity of each phase, respectively. The subscripts *si* and *g* denote the *i*th ($i = 1,2$) component of the solids phase and gas phase, respectively. The sum of the volume fractions of each phase equals to unity:

$$\alpha_g + \sum_{i=1}^{2}\alpha_{si} = 1 \tag{3}$$

The equations of momentum balance for the gas and the solid phase *si* ($i = 1, 2$) are

$$\frac{\partial}{\partial t}(\alpha_g\rho_g\vec{v}_g) + \nabla\cdot(\alpha_g\rho_g\vec{v}_g\vec{v}_g) = -\alpha_g\nabla p + \nabla\cdot\overline{\overline{\tau_g}} + \sum_{i=1}^{2}K_{sig}(\vec{v}_{si} - \vec{v}_g) + \alpha_g\rho_g\vec{g} \tag{4}$$

$$\frac{\partial}{\partial t}(\alpha_{si}\rho_{si}\vec{v}_{si}) + \nabla\cdot(\alpha_{si}\rho_{si}\vec{v}_{si}\vec{v}_{si}) = -\alpha_{si}\nabla p - \nabla p_{si} + \nabla\cdot\overline{\overline{\tau_{si}}} + K_{gsi}(\vec{v}_g - \vec{v}_{si}) + K_{sjsi}(\vec{v}_{sj} - \vec{v}_{si}) + \alpha_{si}\rho_{si}\vec{g}, \quad \text{for } i \neq j \tag{5}$$

respectively; where p and p_{si} are the gas phase pressure and the solids phase pressure of *i*th component, respectively; $\vec{g}$ represents the gravitational acceleration; K_{sig}, K_{gsi} and K_{sjsi} are the inter-phase interaction terms.

The solids pressure consists of a kinetic term and a collision term:

$$p_{si} = \alpha_{si}\rho_{si}\theta_{si} + 2\frac{d_{sjsi}^3}{d_{si}^3}(1+e_{sjsi})\alpha_{si}\alpha_{sj}\rho_{si}g_{0,sjsi}\theta_{si} \tag{6}$$

where e_{ss} is the particle restitution coefficient; θ_{si} denotes the granular temperature; d_{si} is the particle diameter of the *i*th component; d_{sjsi} is the average diameter of the *i*th solids component and the *j*th solids component. $g_{0,ss}$ represents the solids radial distribution function. $g_{0,sisi}$ and $g_{0,sisj}$ are calculated by

$$g_{0,sisi} = \left[1 - \left(\frac{\alpha_s}{\alpha_{s,max}}\right)^{\frac{1}{3}}\right]^{-1} + \frac{1}{2}d_{si}\sum_{i=1}^{2}\frac{\alpha_{si}}{d_{si}} \tag{7}$$

and

$$g_{0,sisj} = \frac{d_{si}g_{0,sjsj} + d_{sj}g_{0,sisi}}{d_{si} + d_{sj}} \tag{8}$$

respectively, with

$$\alpha_s = \sum_{i=1}^{2}\alpha_{si} \tag{9}$$

In Eqs. (4) and (5), $\overline{\overline{\tau_g}}$ and $\overline{\overline{\tau_{si}}}$ are the stress–strain tensors of the gas phase and the solids phase, respectively:

$$\bar{\bar{\tau}}_g = \alpha_g\mu_g[\nabla\vec{v}_g + \nabla\vec{v}_g^T] - \frac{2}{3}\alpha_g\mu_g\nabla\cdot\vec{v}_g\vec{I} \tag{10}$$

$$\bar{\bar{\tau}}_{si} = \alpha_{si}\mu_{si}[\nabla\vec{v}_{si} + \nabla\vec{v}_{si}^T] + \alpha_{si}\left(\lambda_{si} - \frac{2}{3}\mu_{si}\right)\nabla\cdot\vec{v}_{si}\vec{I} \tag{11}$$