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Overestimation of the interface thickness around convex-shaped grain by sectional analysis

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Abstract

Sectional analysis is commonly used to characterize qualitatively and quantitatively the interfacial transition zone (ITZ) of concrete, especially its thickness. However, the normal of the sectioning plane is in general not perpendicular to the normal of the aggregate surface due to the irregularity of the shape of the aggregates. Therefore, the sectional approach overestimates ITZ thickness. Calculating the degree of overestimation is key to obtain a precise measure of ITZ thickness and therefore the influence of the ITZ on the macro-properties of composites. However, the irregularity of the aggregates makes this a difficult task. In this contribution, some related theorems and results from geometrical probability were used to study the relationship between the statistical value of the apparent ITZ thickness and actual ITZ thickness. General solutions are derived for an arbitrary convex-shaped aggregate grain for both two- and three-dimensional cases.

Keywords: Section analysis method; Interface thickness; Shape of grain; Overestimation; Geometrical probability

1. Introduction

Composites, such as concrete, consist of a matrix and reinforced inclusions (grains or fibers). The interfacial transition zone (ITZ) between matrix (e.g. cement paste) and reinforced inclusions (aggregate grains) is the weakest part of composites [1,2]. Due to the weakness of the ITZ, many researchers have studied the influence the proportions of raw materials and the mixing process, etc., on the ITZ microstructure so as to improve the macro-properties of composites. In order to quantify the influence of the ITZ on the macro-properties of composites, some researchers have derived a formula to calculate the volume fraction of the ITZ [3,4]. However, because the constituents of most composites are opaque, sectional analysis has been commonly used to study the ITZ microstructure. The thickness of the ITZ is an important parameter characterizing the nature of the ITZ. However, the shape of an aggregate grain is normally irregular in concrete. Hence it is obvious that the apparent thickness of the interface obtained from the random sectional plane is greater than the actual value because the normal of the sectional plane is seldom perpendicular to the normal of the surface of inclusion. This phenomenon has seldom been incorporated into models in the literature. In order to provide correct information on the ITZ microstructure, it is necessary to calculate the degree of overestimation between the apparent ITZ thickness and the actual ITZ thickness from a statistical point of view. However, this is not an easy task due to the irregularity of shape of the aggregate grain in concrete. Scrivener [5] mentioned a factor of 1.23 for the ratio between the apparent ITZ thickness and the real value in three-dimensional (3-D) systems, but no detail is available on how this value was obtained. Stroeven [6] proposed a partial analytical solution. For spherical aggregate particles, he studied the

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ratio of the statistical mean apparent ITZ thickness to the real value. Later, this problem was further studied for two-dimensional (2-D) circular, rectangular and elliptical particles [7]. The results showed that the shape of an aggregate had an effect on the degree of overestimation. For other regular or irregular particles, no relevant results are available from the literature. In this contribution, some theorems and results of geometrical probability are used to derive the analytical solution for the overestimation of the ITZ thickness around 2-D or 3-D aggregate grains of arbitrary shape.

2. Mathematical description of physical problem

For convenience, the following assumptions are made:

1. Concrete is composed of three phases: aggregate grains, ITZ and bulk paste. Each aggregate grain is surrounded by an ITZ with a fixed thickness, t. That is to say, the aggregate grain surface is concentric with the ITZ layer. The remaining volume of concrete is filled with bulk paste.
2. The surface spacing between adjacent aggregate particles is large enough to exclude interference of the ITZ microstructures between neighboring particles.
3. The region occupied by an aggregate particle is a bounded closed convex region.
4. The random sectional plane occurs with equal probability in any position and direction.

A random sectional plane through the aggregate grain is equivalent to a family of parallel lines in this plane. Therefore, the mean apparent ITZ thickness obtained by randomized sectional plane methods is the same as the statistical mean intercept length of a random line intersecting a convex ring (for the 2-D case) or a convex shell (for the 3-D case). Thus, the problem of assessing the mean apparent ITZ thickness can be converted into the following mathematic problem.

The bounded closed convex region occupied by the aggregate particle is called the convex set $\mathbf{K_1}$. The bounded closed convex region occupied by both aggregate grain and ITZ paste is called the convex set $\mathbf{K_2}$. Then, the set of ($\mathbf{K_2}$–$\mathbf{K_1}$) is the region occupied by the ITZ, and is denoted as a convex ring (for 2-D cases) or a convex shell (for 3-D cases). Therefore, the statistical mean apparent ITZ thickness can be calculated by the following steps: (1) calculate the mean intercept length L_1 between infinite straight line and convex set $\mathbf{K_1}$; (2) calculate the mean intercept length L_2 between infinite straight line and convex set $\mathbf{K_2}$; (3) replace L_2 by L_2' before calculating the mean apparent ITZ thickness t' (the reason for this is shown below); (4) obtain the generalized analytical formula for t'; (5) build the generalized analytical formula between the mean apparent ITZ thickness and the actual ITZ thickness.

The objects in real concrete are 3-D. However, due to the opacity of concrete, sectional analytical is commonly used to study the microstructure. Therefore, the overestimation of the ITZ thickness will first be calculated for 2-D aggregate particles, then further analyzed for 3-D aggregate particles with any convex shape. This is not only for the purpose of mathematical completeness but also because of the popularity of sectional analysis in the field of materials sciences. Furthermore, we want to show the difference in ratio of t'/t between the 2-D and the 3-D analysis.

3. 2-D aggregate particles

According to the aforementioned assumptions, the region occupied by the 2-D aggregate particles is a region of a bounded closed convex set $\mathbf{K_1}$. Assuming that the area of convex set $\mathbf{K_1}$ is A_1, the perimeter of the boundary curve of $\mathbf{K_1}$ is C_1. The bounded closed convex set $\mathbf{K_2}$ is obtained by expanding $\mathbf{K_1}$ a distance t normal to the boundary of $\mathbf{K_1}$ in all directions. (In geometrical probability terminology, $\mathbf{K_2}$ is the Minkowski sum of $\mathbf{K_1}$ with a small (Euclidean) ball of radius t.) Therefore, $\mathbf{K_1}$ and $\mathbf{K_2}$ are a pair of concentric convex sets and $\mathbf{K_2}$ is a parallel set of $\mathbf{K_1}$. The set ($\mathbf{K_2}$–$\mathbf{K_1}$) stands for the ITZ of concrete. Let A_2 be the area of convex set $\mathbf{K_2}$ and C_2 be the perimeter of the boundary curve of $\mathbf{K_2}$.

Now, put the region occupied by an aggregate grain and the ITZ into the 2-D orthogonal Cartesian coordinate system. Then, let a family of straight lines parallel to one axis, such as the y-axis, pass through the region of $\mathbf{K_1}$ (in Fig. 1). Assuming that the shadow length of projecting $\mathbf{K_1}$ onto the x -axis is $l'_{\mathbf{K_1},\perp X}$, the area of $\mathbf{K_1}$ is then obtained by integrating successively the length of each intercept along the $l'_{\mathbf{K_1},\perp X}$. Therefore, according to Cauchy's theorem [8], the average intercept length along the y-axis, L_Y, is:

$$L_Y = A_1 / l'_{\mathbf{K_1},\perp X}. \quad (1)$$

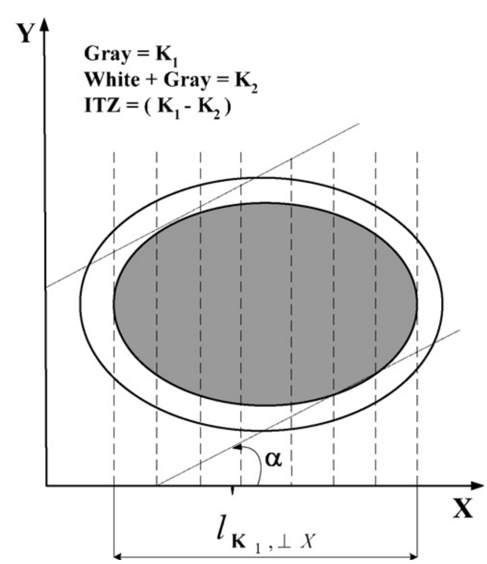

Fig. 1. Diagram of average intercept length of a family of parallel lines along the y-axis with 2-D convex-shape aggregate.