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Fiber volume fraction and ductility index of concrete beams

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ABSTRACT

The mechanical response of fiber-reinforced concrete (FRC) beams depends on the amount of fibers, and the transition from brittle to ductile behavior in bending is related to a critical value of fiber volume fraction. Such quantity, which is mechanically equivalent to the minimum amount of steel rebars in reinforced concrete beams, can be defined according to the new approach proposed herein. It derives from the application of a general model and from the introduction of the so-called ductility index (*DI*). When FRC beams show a ductile behavior *DI* is positive, whereas *DI* is negative in the case of brittle response. Both the theoretical and experimental results prove the existence of a general linear relationship between *DI* and the fiber volume fraction. Accordingly, a new design-by-testing procedure can be used to determine the critical value of fiber volume fraction, which corresponds to a ductility index equal to zero.

1. Introduction

Depending on the fiber volume fraction V_f used to reinforce the cementitious matrix, fiber-reinforced concrete (FRC) ties behave differently (Fantilli et al. [1], Naaman [2]). Specifically, in the case of low values of V_f, the tensile force F after cracking (which occurs at the cracking load F_{cr}), remains lower than F_{cr} as the elongation ΔL increases (see Fig. 1a). Such ties fail in a brittle manner, and the so-called strain-softening occurs in the presence of a single crack. Conversely, with an higher amount of fibers, and after the softening stage subsequent to the growth of the first crack, F increases and reaches the ultimate load $F_u > F_{cr}$. This is the case of the ductile response, in which the ties show strain-hardening behavior and multiple cracking. At the transition from the brittle to the ductile response, when $F_u = F_{cr}$ (Fig. 1a), the corresponding critical value of V_f can be considered as the minimum amount of fibers that guarantees the strain-hardening behavior of the FRC ties (Fantilli et al. [1]).

Similarly, the behavior of FRC beams in three point bending, as illustrated in Fig. 1b in terms of applied load P *vs.* midspan deflection δ, is also a function of V_f (Naaman [2]). At the first relative maximum (when $P = P_{cr*}$), the effective cracking takes place, whereas at ultimate load P_u (the second relative maximum) strain localization occurs in the tensile zone. The value of P_{cr*} is in turn higher than the first cracking load (i.e., P_{cr}), which corresponds to the attainment of the tensile strength in the bottom edge of the FRC beam (Fig. 1b). For low amounts of fibers, P_u is always lower than P_{cr*}, and the brittle response of the beams (called deflection-softening) is evidenced by the presence of a single crack. Conversely, both the deflection-hardening (i.e., $P_u > P_{cr*}$ in Fig. 1b), and the presence of more than one crack as well, are the results of tests performed on FRC beams containing high amounts of fibers. Consequently, at the transition from brittle to ductile behavior, when $P_u = P_{cr*}$ in Fig. 1b, the minimum ductility is attained and a critical amount of fibers can be defined. As this quantity of fibers has the same mechanical role of the minimum area of steel reinforcing bars in reinforced concrete beams (Fantilli et al. [3]), it can be defined as the minimum amount of fiber-reinforcement (i.e., $V_f = V_{f,min}$ in Fig. 1b).

A univocal and simple approach able to predict $V_{f,min}$ in FRC beams does not exist, despite the huge number of tests available in the current literature. For instance, Naaman [2] proposed a formula to compute $V_{f,min}$ based on the equation $P_{cr*} = P_u$, where the values of both the loads are functions of the flexural and of the residual tensile strengths of FRC, respectively. However, the relationship between the strengths and the fiber volume fraction cannot be easily and univocally defined, hence the analytical prediction of $V_{f,min}$ is not always effective. On the other hand, the approaches suggested by code rules are even more complicated, due to the

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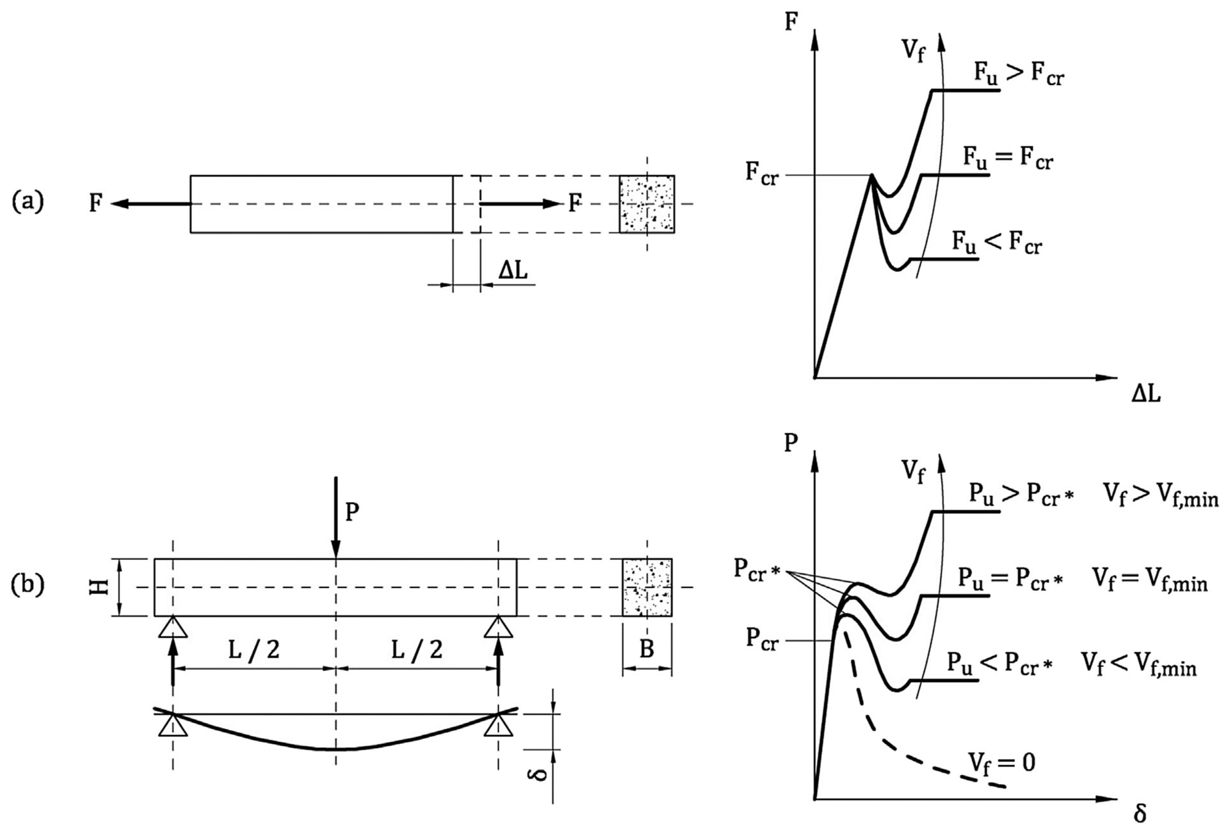

Fig. 1. The behavior of FRC structures as a function of the fiber content: (a) axial load *vs.* elongation diagram of a tie; (b) applied load *vs.* midspan deflection of a beam in three point bending.

large experimental campaigns required for the definition of $V_{f,min}$. In particular, Model Code 2010 (*fib* [4]) firstly recommends the classification of FRC, and the evaluation of the residual strengths, by means of three point bending tests on notched beams. Then, the displacements measured in a second series of tests, performed on full-scale FRC elements in bending, with different contents of fibers, are needed. The ductility requirement in bending (and the corresponding $V_f \geq V_{f,min}$) is satisfied when the ultimate or the peak displacements are sufficiently large (Caratelli et al. [5], de la Fuente et al. [6]).

With the aim of simplifying the evaluation of $V_{f,min}$, a new design-by-testing procedure, capable of predicting the brittle/ductile behavior of FRC beams in bending, is proposed in the following. It is the result of both the theoretical and experimental investigations described in the next sections.

2. General model

A multi-scale general model is introduced herein to predict the behavior of the FRC beam depicted in Fig. 1b. The fiber-reinforcement is modeled with an ideal tie (Fig. 2a), composed by a straight fiber and the surrounding cementitious matrix, having a single orthogonal crack in the midsection. The pullout mechanism of this element provides the stress-strain relationship of the cracked FRC. Only when this relationship is known, can the mechanical response of the FRC beams in bending be properly defined.

2.1. Modeling the fiber pullout

The ideal tie illustrated in Fig. 2a has a square cross-section, in which the area of the cementitious matrix A_c is a function of the amount of fibers used in the FRC beam:

$$A_c = \frac{A_f}{V_f} = \frac{\pi \cdot \phi^2}{4 \cdot V_f} \tag{1}$$

where A_f, ϕ = area and diameter of the fiber cross-section, respectively.

The portion of the tie delimited by the cracked cross-section (in the midspan) and the so-called Stage I cross-section (where the perfect bond between steel and concrete is re-established) is investigated. Within this block of length l_{tr} (= transfer length), as the horizontal coordinate z increases, stresses move from steel to concrete in tension, due to the bond-slip mechanism acting at the interface of the materials. Such slip s vanishes in the Stage I cross-section (Fig. 2b), where stresses (of fiber $\sigma_{f,I}$ and of concrete $\sigma_{c,I}$ in Fig. 2c) are computed with the well-known linear elastic formulae, under the hypothesis of perfect bond between the materials:

$$\sigma_{f,I} = n \cdot \frac{N}{A_c + n \cdot A_f} \tag{2}$$

$$\sigma_{c,I} = \frac{N}{A_c + n \cdot A_f} \tag{3}$$

where $n = E_f/E_c$ = ratio between the Young's moduli of the fiber and of the cementitious matrix; N = axial load applied to the ideal tie (Fig. 2a).

Within the transfer length l_{tr}, the interaction between fiber and matrix is described by the following equilibrium and compatibility