Cement & Concrete Composites 48 (2014) 42–52

Contents lists available at ScienceDirect

Cement & Concrete Composites

journal homepage: www.elsevier.com/locate/cemconcomp

# Simulation of single fiber pullout response with account of fiber morphology

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ARTICLE INFO

*Article history:*
Received 2 January 2012
Received in revised form 2 January 2014
Accepted 9 January 2014
Available online 19 January 2014

*Keywords:*
Cementitious composites
Fiber morphology
Finite element analysis
Twisted fibers

ABSTRACT

A finite element model was developed at the single fiber length scale to predict the quasi-static pullout response of individual fibers from cementitious composites. The model accounts for energy dissipation through granular flow of the interfacial transition zone (ITZ) and matrix, plastic work in the fiber, and frictional dissipation at the fiber–ITZ interface. The considered fiber morphology was a triangular cross section that had been uniformly twisted along the fiber length. The model was calibrated to published experimental data for fiber pitches of 12.7 and 38.1 mm/revolution pulled from cement mortar with a 44-MPa unconfined compressive strength. The model was used to investigate slip-hardening behavior, tunneling of the cement mortar, in situ pullout behavior of helically twisted fibers at a crack plane, and provide an alternate explanation for the pullout response of twisted fibers from a 84-MPa unconfined compressive strength matrix containing silica fume. Calculations show that twisted fibers can provide up to 5 times the peak pullout force and 10 times the total work compared with straight fibers and infer work-hardening behavior during fiber pullout. The findings indicate that the tailoring of fiber morphology and control of constituent properties are important avenues for achieving significant improvements in the performance of fiber-reinforced cementitious composites.



## 1. Introduction

The granular nature of concrete elicits a characteristic quasi-brittle behavior in tension. This undesirable behavior is often mitigated by embedding steel rebar within concrete; however, steel rebar within concrete presents new problems. For example, steel rebar reinforcement is relatively labor intensive to install, susceptible to corrosion through chloride ion transport, and unable to arrest cracks prior to the crack intersecting the steel rebar. Furthermore, the performance of rebar reinforced composites is highly susceptible to the placement of the rebar within the composite. An alternate reinforcement approach is to replace steel rebar with short, discontinuous fibers, resulting in Fiber Reinforced Cementitious Composites (FRCCs). Although they exhibit ductility and toughness [1], FRCCs' limitations in tensile loading require further investigation.

The tensile response of FRCCs depends on six factors: fiber volume fraction, fiber orientation, fiber shape, fiber material properties, cementitious material properties, and properties at the fiber–matrix interface. For steel fibers, it is desired to reduce the fiber volume fraction due to the relative expense of steel fibers compared to the cementitious matrix. Furthermore, fiber volume fractions greater than 2–4%, depending on the fiber's length to diameter ratio, may introduce porosity and fiber clumping during mixing [2]. Assuming a random orientation for short discontinuous fibers within a structure, the other four factors provide avenues for improving the tensile response of FRCCs.

Tensile responses of FRCCs at the mesoscale have been characterized by either direct tension tests of dog bone specimens (cf. Kim et al. [3]) or the flexural bending tests, defined by the American Society for Testing and Materials (ASTM) C1609 testing standard [4]. Both tests allow researchers to infer structure–property–performance relations of FRCCs by systematically changing the structure and material properties of the constituents. For example, Kim et al. [3] reported that FRCCs containing a 2% volume fraction of twisted fibers, defined as a fiber with a polygonal cross section that has been twisted along its primary axis, had between 25% and 49% greater mean first cracking strengths in direct tension tests than FRCCs containing 2% hooked fibers for matrices with unconfined compressive strengths between 28 and 84 MPa. Results for flexural bending tests indicate similar dependencies on the shape of the fiber. For example, Kim et al. [5] showed that FRCCs containing 1.2% fiber volume fractions of twisted fibers had a

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http://dx.doi.org/10.1016/j.cemconcomp.2014.01.003
0958-9465/

**Nomenclature**

| | |
|---|---|
| $\underset{\sim}{\alpha}$ | back stress tensor |
| $\underset{\sim}{\alpha}^{dev}$ | deviatoric back stress tensor |
| $A_f$ | cross-sectional area of fiber |
| $\beta$ | internal friction angle |
| $\beta_i$ | internal friction angle – ITZ |
| $\beta_m$ | internal friction angle – matrix |
| $c$ | constant |
| $C$ | material constant - fiber |
| $d$ | cohesion under pure shear |
| $\varphi_e$ | equivalent fiber diameter |
| $\underset{\sim}{D}^p$ | plastic part of the rate of deformation tensor |
| $\tilde{\varepsilon}^{pl}$ | equivalent plastic strain – fiber |
| $\underset{\sim}{\dot{\varepsilon}}^{pl}$ | plastic strain tensor |
| $\dot{\tilde{\varepsilon}}^{pl}$ | equivalent plastic strain rate – fiber |
| $\dot{\bar{\varepsilon}}^{p}$ | equivalent plastic strain rate |
| $E_i$ | elastic stiffness – ITZ |
| $E_f$ | elastic stiffness – fiber |
| $E_m$ | elastic stiffness – matrix |
| $F$ | yield condition |
| $\underset{\sim}{F}$ | deformation gradient tensor |
| $\underset{\sim}{F}^e$ | elastic deformation gradient tensor |
| $\underset{\sim}{F}^n$ | inelastic deformation gradient tensor |
| $f_i$ | unconfined compressive strength – ITZ |
| $f_m$ | unconfined compressive strength – matrix |
| $G$ | flow potential |
| $\gamma$ | material parameter – fiber |
| $\dot{\gamma}_i$ | shearing rate ($i = 1, 2$) |
| ITZ | interfacial transition zone |
| $\underset{\sim}{I}$ | 2nd rank identity tensor |
| $K$ | ratio of yield stress in triaxial tension to yield stress in triaxial compression |
| $K_i$ | ratio of yield stress in triaxial tension to yield stress in triaxial compression – ITZ |
| $K_m$ | ratio of yield stress in triaxial tension to yield stress in triaxial compression – matrix |
| $\dot{\lambda}$ | plastic multiplier |
| $L_e$ | fiber embedded length |
| $L_{free}$ | fiber free length |
| $\nu_i$ | Poisson's ratio – ITZ |
| $\nu_f$ | Poisson's ratio – fiber |
| $\nu_m$ | Poisson's ratio – matrix |
| $\mu$ | coefficient of coulomb friction |
| $p$ | hydrostatic pressure |
| $p_{contact}$ | contact pressure at interface |
| $q$ | mises equivalent stress |
| $\rho_i$ | mass density – ITZ |
| $\rho_f$ | Mass density - fiber |
| $\rho_m$ | mass density – matrix |
| $r$ | third invariant of deviatoric stress |
| $\sigma^o$ | yield stress – fiber |
| $\underset{\sim}{\sigma}$ | cauchy stress tensor |
| $\underset{\sim}{S}$ | deviatoric stress |
| $\tau_{eq}$ | equivalent frictional stress |
| $\tau_i$ | shear stress at interface between two different materials ($i = 1, 2$) |
| $\tau_{crit}$ | critical frictional stress |
| $t$ | extended Drucker–Prager stress in meriodonal plane |
| $\underset{\sim}{u}$ | displacement vector |
| $\underset{\sim}{x}$ | deformed coordinate system |
| $\underset{\sim}{X}$ | fixed reference coordinate system |
| $\psi_i$ | dilation angle – ITZ |
| $\psi_m$ | dilation angle – matrix |

13% greater mean modulus of rupture than FRCCs containing hooked fibers. Flexural bending tests reported by Soroushian and Bayasi [6] reported that FRCCs containing 2% volume fraction straight smooth fiber had a 35% reduction in modulus of rupture as compared to FRCCs containing 2% volume fraction off hooked fibers with a similar length and diameter. Clearly, the geometry of a fiber influences the tensile properties of FRCCs.

To understand why one fiber geometry is more effective than another, single fiber pullout tests are used to characterize the pullout responses of fibers with different morphologies. As reported by many researchers (e.g., Easley et al. [7]; Kim et al. [3]; Boshoff et al. [8]; Cunha et al. [9]), a single straight, smooth fiber pulled in the axial direction from a cementitious matrix exhibits three common energy storage and dissipation stages: (1) an initial elastic storage stage, in which the fiber undergoes relatively small displacements before the peak force is reached; (2) a debonding stage, in which the chemical bonds between the fiber and cementitious matrix break, resulting in a drop in force; and (3) a friction-dominated stage, in which the pullout force decreases monotonically as the fiber pulls out. For straight, smooth fibers, the fiber length has a strong influence on the peak pullout force. For example, Cunha, Barros, and Sena-Cruz [9] reported a 100% increase in peak pullout force when fiber length is increased from 20 mm to 30 mm. Additional studies were conducted by Chan and Chu [10] and Guerrero and Naaman [11] to determine the effects of matrix constituents on pullout behavior.

Analytical models of a single straight, smooth fiber being pulled out of a matrix have been framed in terms of energy balance [12] and equilibrium [13]. The equilibrium-derived analytical model uses experimental data to determine five constants: bond modulus, bond strength, constant frictional bond stress, and two decaying frictional parameters. Numerically, Li and Mobasher [14] used a two-dimensional axisymmetric framework containing three linear elastic constitutive relations to simulate the three pertinent materials: fiber, interface, and matrix. The modeled mechanisms include fiber debonding and friction. A clamping pressure was applied at the outer edge of the matrix to simulate shrinkage. Results were presented and compared to experimental data for the first 0.1 mm of end slip.

Hooked fibers exhibit behaviors different from those of straight fibers. As reported by Cunha et al. [9], a hooked fiber embedded 20 mm into a matrix shows a peak pullout force approximately 4.5 times that of a straight, smooth fiber embedded at the same depth. Even though the peak pullout force of a hooked fiber increases with the embedded length of the fiber, the increase is not as pronounced as that for straight, smooth fibers [9]. In addition to the three energy storage and dissipation mechanisms of straight, smooth fibers, hooked fibers also dissipate energy via plastic work during pullout. Although not a distinct mechanism, the residual stress at a fiber's hook appears to increase normal tractions and ultimately the force required during the friction-dominated stage of pullout.

An analytical model to predict the pullout force versus end slip relation for hooked fibers was introduced by Alwan et al. [15], who extended the model of straight, smooth fibers given by Naaman et al. [13]. The model predicts four different characteristic