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Axial stress–strain relationship for FRP confined circular and rectangular concrete columns

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Abstract

A general mathematical model is developed to describe the stress–strain (f_c–ε_c) relationship of FRP confined concrete. The relationship is applicable to both circular and rectangular columns, and accounts for the main parameters that influence the stress–strain response. These include the area and material properties of the external FRP wraps, the aspect ratio of rectangular column sections, the corner radius used for FRP application, and the volumetric ratio and configuration of internal transverse steel. The proposed model reproduced accurately experimental results of stress–strain or load–deformation response of circular and rectangular columns. In addition to its importance in evaluating the effect of FRP confinement on the ultimate axial strength of concrete columns, the developed f_c–ε_c relationship can be employed very efficiently and effectively for analyzing the response of FRP confined concrete under different types of load application.

Keywords: Columns; Confined concrete; Ductility; Fiber reinforced polymers; Stress; Strain

1. Introduction

Several experimental studies have been conducted for evaluating the axial strength characteristics of concrete columns confined externally with fiber reinforced polymer (FRP) composites. These studies have identified most of the critical parameters that influence the axial strength of FRP confined columns [1]. These include the area and material properties of the transverse FRP reinforcement, arrangement of reinforcement, type of column section (rectangular, circular), the aspect ratio of rectangular section, and the radius of the section corner prepared for FRP application. Although most of these parameters are identical to those that influence the stress–strain response of steel confined concrete, because steel behaves in elasto-plastic manner while FRP is a linear elastic material, the axial strength and stress–strain behavior for concrete confined with FRP composites are substantially different as compared to concrete confined with steel ties.

Most of the available studies on the axial strength characteristics of FRP confined columns have concentrated on circular columns, while relatively very few addressed rectangular columns [2,3]. Similar to the behavior of steel confined concrete [4], lateral confinement of rectangular sections using FRP, particularly those with large aspect ratio, is not as effective as circular sections [5]. Unlike circular columns where the full column section is confined, rectangular columns need sizable axial strain before the flat sides are able to mobilize the FRP confinement pressure. According to ACI Committee 440 [1], confining square or rectangular columns with FRP jackets can provide marginal increase in the axial load capacity, but because of the many unknowns associated with this type of application, it is not possible with the current state of knowledge to provide recommendations on the use of FRP for strengthening rectangular columns. Furthermore, because of the substantial number of parameters involved, very

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Nomenclature

A_{frp}	area of transverse FRP reinforcement
A_{fa}	area of longitudinal FRP reinforcement
A_g	gross area of section
A_{cc}	area of concrete core
A_e	area of effectively confined concrete
A_s	area of column longitudinal reinforcement
b	section width
D	diameter of circular section
d_s	diameter of spiral or hoop
E_c	modulus of elasticity of concrete
E_f	modulus of elasticity of transverse FRP
E_{fa}	modulus of elasticity of longitudinal FRP
E_{lf}	lateral modulus of elasticity of FRP
E_{ls}	lateral modulus of elasticity of steel
E_s	modulus of elasticity of steel
f_c	concrete stress
f'_c	compressive strength of unconfined concrete
f_{cc}	stress in confined concrete
f'_{cc}	compression strength of confined concrete
f_{co}	stress at the intersection point between the 1st and 2nd stage of the stress–strain curve
f_{cu}	stress corresponding to a limiting strain ε_{cu}
f_ℓ	effective lateral confining pressure
f'_ℓ	hydrostatic confining pressure
f_s	steel stress
f_y	yield stress of longitudinal column reinforcement
f_{yt}	yield stress of transverse steel ties or hoops
h	section depth
k_1	confinement effectiveness coefficient
k_e, k_v	confinement effectiveness parameters
n_f	number of transverse FRP layers
P	applied axial load
r	corner radius
s'	clear spacing between transverse hoops or spirals
t_f	thickness of one FRP layer
w	clear distance between adjacent longitudinal bars
w_{xi}, w_{yi}	the *i*th clear distance between adjacent longitudinal bars along the horizontal *x*- and *y*-dimensions respectively
x, y	concrete core dimensions to center line of peripheral hoop
ε_c	concrete strain
ε_{cc}	concrete strain for confined concrete
ε_{co}	concrete strain at the intersection point between the 1st and 2nd stage of the stress–strain curve
ε_{cu}	limiting concrete strain
ε_{fu}	fracture strain of the FRP
ε_ℓ	lateral concrete strain
$\varepsilon_{\ell o}$	lateral concrete strain at intersection point between the 1st and 2nd stage of the stress–strain curve
ε_o	strain at maximum stress for unconfined concrete
ε_{yt}	yield strain of transverse hoops
ρ_{cc}	steel ratio relative to the concrete core section
ρ_f	volumetric ratio of FRP reinforcement
ρ_s	ratio of column longitudinal reinforcement
ρ_{st}	volumetric ratio of hoop reinforcement

few studies have attempted to generate the stress–strain response of concrete confined with FRP composites taking into account rectangular sections. In evaluating the axial–flexural capacity of concrete columns confined with FRP straps, Saadatmanesh et al. [6] adopted the stress–strain model of Mander at al. [4] which was developed for concrete confined with ordinary steel. However, as pointed out by Mirmiran and Shahawy [7], given the significantly different mechanical properties of the steel and FRP, extending confinement models developed originally for steel to cover FRP confined columns may not be appropriate. A stress–strain model for FRP confined concrete was developed by Toutanji [8] but it is applicable mainly for circular columns.

In this study, a comprehensive and yet simple mathematical model is developed to produce the stress–strain response of FRP confined concrete column sections. In addition to its great importance in predicting the effect of FRP confinement on the axial load capacity of columns, the generation of such a stress–strain relationship is essential for conducting analytical studies of the response of FRP confined concrete under different types of load applications, including axial and flexural loads [9].

2. Confinement models

Most of the available models for evaluating the compression strength and ductility of confined concrete are based on the confinement model derived experimentally by Richart et al. [10,11] using concrete specimens confined with active hydrostatic fluid pressure

$$f'_{cc} = f'_c + k_1 f'_\ell \tag{1a}$$

$$\varepsilon_{cc} = \varepsilon_o\left(1 + k_2\frac{f'_\ell}{f'_c}\right) \tag{1b}$$

where f'_{cc}, ε_{cc} are the compressive strength and corresponding strain of confined concrete; f'_c, ε_o are the compressive strength and corresponding strain for unconfined concrete; f'_ℓ is the lateral hydrostatic pressure; $k_1 = 4.1$, and $k_2 = 5k_1$.

Among the most widely used models to describe the axial strength of reinforced concrete columns confined with