Cement and Concrete Research 79 (2016) 353–365

Contents lists available at ScienceDirect

Cement and Concrete Research

journal homepage: http://ees.elsevier.com/CEMCON/default.asp

A new model for the cracking process and tensile ductility of Strain Hardening Cementitious Composites (SHCC)

Cong Lu *, Christopher K.Y. Leung

Department of Civil and Environmental Engineering, HKUST, Clear Water Bay, Kowloon, Hong Kong

A R T I C L E   I N F O

Article history:
Received 12 December 2014
Accepted 19 October 2015
Available online 4 November 2015

Keywords:
SHCC
Fiber reinforcement (E)
Micromechanics (C)
Modeling (E)

A B S T R A C T

Strain Hardening Cementitious Composites (SHCC) are materials exhibiting tensile hardening behavior up to several percent strain accompanied by the formation of fine multiple cracks. Their tensile ductility is governed by the spacing and opening of cracks, which depend on the stress transfer between the fibers and the matrix. In this article, a new analytic model which takes into consideration the effects of non-uniform matrix strength, post-cracking increase in fiber bridging stress and fiber rupture on stress transfer and multiple cracking behavior of SHCC is developed. Using material parameters within the range reported in the literature, simulation results can reach reasonable agreement with test data on SHCC for two different fiber contents. The effect of fiber length on tensile behavior of SHCC is then simulated to illustrate the applicability of the model to material design. The new model should be helpful to the micromechanics-based design of SHCC for various ductility requirements.

1. Introduction

The brittleness of cementitious materials is their major limitation. Even with steel reinforcements, the poor resistance to cracking makes cementitious members vulnerable to seismic loading and impact. In addition, the formation and opening of cracks facilitate the penetration of water and chemicals which greatly affects the durability of the structure. To control cracking, fibers can be added. With increasing fiber content, the cementitious composite transforms from a quasi-brittle material, with tension-softening behavior after a crack is formed, into a Strain Hardening Cementitious Composite (SHCC), with hardening behavior up to several percent strain, accompanied by the formation of multiple cracks with tightly controlled openings [1]. With proper design guided by micromechanics, SHCC can be made with moderate fiber content of 2% or less in total volume of materials [1].

The high ductility of SHCC is achieved by the formation of multiple cracks which is possible if the fibers can carry a higher tensile force than that corresponding to the first cracking of the cementitious matrix. After the first crack is formed, the applied tension can continue to increase. At the crack, the matrix stress drops to zero, and the stress in the bridging fibers has to increase to maintain equilibrium. Away from the crack plane, the additional stress taken by fibers will be transferred back to the surrounding matrix through the fiber/matrix interface. The stress in the matrix increases with distance from the crack, and at a critical transfer distance (x_d), the stress reaches the matrix strength again. At any distance larger than x_d from a certain crack, another crack can form. This is the mechanism for multiple cracking to take place. After all the cracks are formed, the minimum crack spacing is x_d while the maximum spacing is $2x_d$ [2].

For the hardening composite, the ultimate tensile strength is reached when the fiber bridging stress reaches its maximum value. The corresponding ultimate tensile strain is governed by the crack spacing. Theories for crack spacing calculation in fiber reinforced brittle matrix composites have been established in the 1970s [2–4] and further developed for SHCC in the 80s [5,6] and 90s [7,8]. In all existing theories, the cracking strength of the composite is assumed to be uniform along the whole member. Under this assumption, once the cracking strength of composites is reached, all cracks will form simultaneously and the value of x_d is the same for each crack. This is inconsistent with experimental observation where multiple cracks form in sequence with increasing load. In reality, the cracking strength at different cross-sections along a member varies, and the plausible reasons include: 1) variation of flaw size in each cross-sectional plane, 2) variation of fiber volume fraction in each plane, and 3) interaction between matrix cracks [9]. As x_d is the distance from a crack for the matrix strength to be reached again, when cracks form sequentially with increasing load, the value of x_d also varies. To properly consider this effect in the calculation of crack spacing, x_d can be derived in terms of the crack opening displacement (COD) which has a one-to-one relation to the crack bridging stress in the rising part of the stress vs COD curve of the fiber composite.

In practice, although fiber usually has higher tensile strength than cementitious matrix, fiber rupture often occurs at high tensile stress after matrix cracks [10–12]. The rupturing of fibers clearly has an effect on the crack bridging stress as well as x_d, because stress transfer becomes less effective with a smaller number of load carrying fibers. However, this was not considered in existing theories for crack spacing.

* Corresponding author. Tel.: +852 64894232.
E-mail address: cluab@ust.hk (C. Lu).

http://dx.doi.org/10.1016/j.cemconres.2015.10.009
0008-8846/

In this paper, with the assumption of non-uniform matrix first-cracking strength along the member, a new model to predict the crack formation (with non-uniform spacing) in SHCC is proposed. Knowing the number and opening of the cracks, the stress–strain relation is also derived. In the model, the combined effects of increasing COD and fiber rupture on x_d are considered. The simulated stress–strain relations will be compared with experimental results. Also, numerical simulation is carried out with the model to study the effect of fiber length on stress–strain behavior. The ultimate goal of this study is to establish a framework to guide the design of SHCC with specified tensile ductility through the proper simulation of multiple cracking.

2. Existing theories on transfer distance

As the new model to be developed is an extension of existing theories, a brief review of existing work is carried out first. The tensile ductility of a composite is dependent on the crack spacing. At saturated cracking (i.e., when the number of cracks stops increasing), the final crack spacing is theoretically between x_d and $2x_d$. Based on the analogy with minimum average spacing between cars of length x_d and parked randomly along an infinite line, the average crack spacing was derived as $1.337x_d$ [13]. Neglecting the elastic deformations between cracks, the tensile ductility, which is the strain at maximum stress, can be calculated [14]:

$$\varepsilon = \frac{\delta_{peak}}{1.337x_d} \quad (1)$$

where δ_{peak} is the crack opening at peak strength.

The transfer distance x_d was first derived by Aveston and Kelly [4] for continuous aligned fibers and then for randomly oriented short fibers in [3]. In the analysis, when a crack occurs in the matrix, the released stress is carried by the bridging fibers crossing the crack plane. The significant increase in fiber strain and decrease in matrix strain (due to unloading) result in debonding and sliding at the fiber/matrix interface. For a short fiber at a given angle to the crack plane, the fiber stress is then transferred back to the matrix through: 1) interfacial friction along the fiber, and 2) a pulley force at the exit point of an inclined fiber [3]. The stress transfer distance x_d required for the matrix strength to be reached again is calculated from [4,15]:

$$F_{friction} + F_{pulley} = \sigma_{mu} V_m. \quad (2)$$

In Eq. (2), σ_{mu} is the matrix strength and V_m is the matrix volume fraction. $F_{friction}$ represents the load per unit area of composite transferred to the matrix by friction along the fiber/matrix interface, which is a function of distance from the crack surface, and F_{pulley} represents the load per unit area transferred to the matrix by the frictional pulley when inclined fibers change angle at the crack surface.

After cracking, the opening of the crack is associated with the elongation of the fiber. For flexible fibers that can be modeled as strings, the elongation (which is the small length of fiber displacing into the space between the crack surfaces) is the same for all angles. The extent of debonding along the fiber would then also be independent of fiber inclination. To see if cracking can occur at a longitudinal distance x_d from the crack, we consider the situation where the length of debonded zone (l_{de}) is also of length x_d so the stress transfer is 'completed' within this length (as there is no more frictional transfer beyond $x = x_d$). The situation is illustrated in Fig. 1a. To calculate $F_{friction}$ at x_d, the bridging fibers are divided into two groups according to embedment length. As shown in Fig. 1, the group of fibers with embedment length l_e larger than x_d will be able to transfer frictional stress back to matrix over the distance of x_d along the fiber. To find the average longitudinal matrix stress in the plane A–A (see Fig. 1) at distance x_d from the crack, the force transferred by each fiber to the matrix is obtained as $\pi d_f x_d \tau \cos\theta$ (where θ is the inclining angle of the fiber) for all fibers with $l_e > x_d$ (Fig. 1a). For fibers with $l_e < x_d$ (Fig. 1b), the force transferred is equal to $\pi d_f l_e \tau \cos\theta$. Because l_e varies from 0 to x_d, the average value is $\pi d_f (l_e/2)\tau\cos\theta$, which is half of that for fibers with $l_e > x_d$.

For fibers randomly distributed in 3-D, the number of fibers crossing any plane at angle between θ and $\theta + d\theta$ to the plane, per unit area of the plane is $N\sin\theta\cos\theta d\theta$ [4], where $N = V_f / \pi r^2{}_f$. V_f is fiber volume fraction. Based on this, integration over the inclining angle which ranges from 0 to 90° can be carried out to obtain the total force (in a direction perpendicular to the crack) per unit area transferred to the matrix at a distance x_d from the crack. The details of the integration can be referred to [2], and the load transferred by friction in this 3-D case is found to be:

$$F_{friciton} = \frac{2V_f \tau x_d (l_f - x_d)}{3r_f l_f} \quad (3)$$

where τ is the interfacial friction between fiber and matrix, and l_f and r_f are the length and radius of fiber respectively.

When a crack is formed and opened, an inclined fiber will exhibit a change in angle at the exit point (Fig. 2). It is assumed by Aveston et al. [2] that the inclined fiber behaves as a string bent over a frictionless pulley. The fiber forces on the two sides of the pulley are then the same. In other words, fibers at different orientations will give the same crack bridging force at the same crack opening. However, experimental results for real composite systems often show an increase of crack bridging force with fiber inclination, which can be explained by the 'snubbing' effect when the fiber (which is modeled as a string) is passing over a frictional pulley [16]. Assuming Coulomb friction at the pulley, it can be shown that the bridging force $P(\theta)$ for a fiber at an inclining angle θ is amplified by a factor of $\exp(f \cdot \theta)$ (Fig. 2) relative to the fiber perpendicular to the crack, where f is the snubbing coefficient to be determined experimentally.

The axial forces at both sides of a single fiber adjacent to the exit point are illustrated in Fig. 3. Before an inclined fiber is subjected to bending and the associated snubbing effect at its exit point, the inner force (defined as the fiber force adjacent to the crack surface on the matrix side) depends only on the embedment length but not the inclining angle. To be more specific, for fibers with larger embedment length that are still in the debonding stage, the inner forces should be the same. For fibers with smaller embedment lengths that are being pulled out, the inner forces will decrease linearly with decreasing embedment length. As the embedment lengths for fibers at different inclinations are following the same uniform distribution from 0 to $l_f/2$, the average inner force of fibers at a certain angle, which is denoted as P_0, should be identical for every angle [7].

Considering force equilibrium in Fig. 3, the force transferred back to matrix along the crack opening direction by a single inclined fiber can be derived as:

$$P_{pulley} = P_0 e^{f\theta} - P_0 \cos\theta. \quad (4)$$

The released stress by matrix at crack plane $\sigma_{mu}V_m$ will be carried by the bridging fibers:

$$\sigma_{mu} V_m = \int_0^{\pi/2} \left(P_0 e^{f\theta}\right) N \cos\theta \sin\theta d\theta. \quad (5)$$

The right-hand side of Eq. (5) represents the integration of fiber bridging force over all inclining angles.

Eq. (5) yields:

$$P_0 = \sigma_{mu} V_m / \left(N \int_0^{\pi/2} e^{f\theta} \cos\theta \sin\theta d\theta\right). \quad (6)$$