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Microprestress–solidification theory of concrete creep: Reformulation and improvement

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ABSTRACT

Concrete creep is strongly affected by the evolution of pore humidity and temperature, which depend on environmental conditions and on the geometry of the concrete member. One advanced model accounting for such phenomena is the extended version of the B3 model referred to as the microprestress–solidification theory. It postulates a differential equation governing microprestress relaxation and generation of additional microprestress caused by changes of temperature and humidity. Here, this equation is rewritten in terms of viscosity, which leads to a reduction of the number of parameters and their easier identification. Based on numerical simulations it is shown that the evolution of mechanical strain measured in creep tests at constant and monotonically increasing temperature can be captured properly by the original microprestress–solidification theory, but certain deficiencies are detected for repeated temperature cycles. An improved version is proposed, identification of new parameters is discussed and good agreement with experimental data is demonstrated.

1. Introduction

In contrast to metals, concrete exhibits creep already at room temperature. This phenomenon results in a gradual but considerable increase in deformation at sustained loads, and needs to be taken into account in the design and analysis of concrete structures.

At sufficiently low stress levels, creep of concrete can be described by models formulated within the framework of linear viscoelasticity with aging. Once the principle of superposition is accepted, the material behavior is uniquely described by the compliance function (or, alternatively, by the relaxation function). The compliance function, J, reflects the time evolution of strain in a creep test at the unit stress level. For aging materials, it is considered as a function of two arguments: time t, which corresponds to the current age of concrete, and time t', which is the age at the beginning of the creep test.

Many analytical expressions and empirical formulae have been proposed in the literature and in the design codes for the approximation of the compliance function based on experimental data and for its prediction from the physical parameters characterizing the concrete and its environment. The present paper focuses on an advanced model which extends the original B3 model [1,2] and uses the concepts of solidification [3,4] and microprestress [5–7]. The main objectives of the paper are to reformulate the governing equations such that a redundant model parameter is eliminated, to develop an efficient numerical algorithm, to validate the model and identify its parameters by fitting experimental data, to point out a certain deficiency in application to cyclic changes of temperature, and to suggest a modification which may eliminate this deficiency.

The creep tests performed by Kommendant et al. [8], Nasser and Neville [9], and Fahmi et al. [10] were used in [7] to validate the MPS theory. References [8] and [9] were focused mainly on creep of sealed concrete specimens subjected to elevated but constant temperatures, and they were simulated by the MPS model in a previous publication of the present authors [11]. Reference [10] studied creep under variable temperature for both sealed and drying specimens, and it is used as a source of experimental data in the present paper, in order to check the performance of the MPS model under repeated temperature cycles. All numerical computations presented here have been performed using the finite element package OOFEM [12,13], with the original MPS model and its modified version implemented by the present authors.

2. Description of the material model

The complete constitutive model for creep and shrinkage of concrete can be represented by the rheological scheme shown in Fig. 1. It consists of (i) a non-aging elastic spring, representing instantaneous elastic deformation, (ii) a solidifying Kelvin chain, representing mainly the short-term creep, (iii) an aging dashpot with viscosity dependent on the microprestress, S, representing the long-term creep, (iv) a shrinkage unit, representing volume changes due to drying, and (v) a unit representing thermal expansion. All these units are connected in series, and thus the total strain is the sum of the individual contributions, while the stress transmitted by all units is the same.

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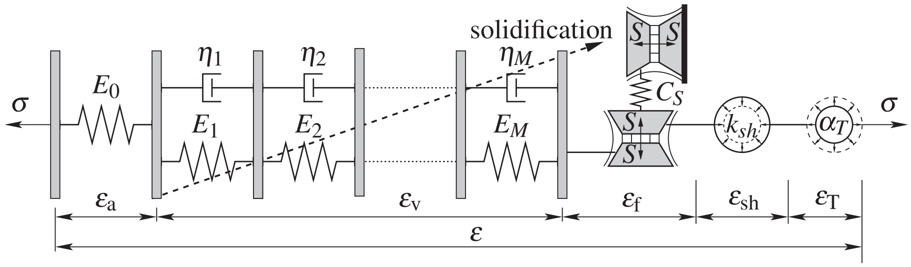

Fig. 1. Rheological scheme of the complete hygro-thermo-mechanical model.

Of course, such an additive decomposition is a model simplification and the actual processes in the meso-structure and micro-structure are more complex. Concrete can be understood as a composite consisting of aggregates embedded as inclusions in cement paste. Aging creep takes place exclusively in the cement paste while aggregates deform elastically. This could be taken into account by sophisticated micromechanical and multiscale approaches which, however, require a higher number of parameters and more detailed information on the properties of individual phases [14–18]. The model discussed here uses a simplified engineering approach and the units in the rheological chain in Fig. 1 should not be interpreted as individual physical constituents.

Detailed justification and calibration of the B3 model were presented in the original papers [1,2]. For instance, the non-aging elastic spring does not represent the classical static elastic modulus, which is certainly age-dependent. It rather reflects the so-called asymptotic modulus, corresponding to extremely short times (even shorter than the times that would correspond to the dynamic modulus). The fact that this unit is considered as non-aging does not follow from theoretical reasoning but from extrapolation of experimental data to extremely short times. Also, the usage of a Kelvin chain for the short-term creep is not dictated by an assumption about serial arrangement of certain physical objects in the microstructure but by the need to represent the compliance function by Dirichlet series, with individual terms corresponding to different parts of the retardation spectrum. One could equally well use a Maxwell chain with parallel arrangement of units, but this would lead to a convenient representation of the relaxation function.

In this paper, attention is focused on the mechanical strain, composed of the first three contributions to the total strain, which are stress-dependent. In the experiments, shrinkage and thermal strains were measured separately on load-free specimens and were subtracted from the strain of the loaded specimen under the same environmental conditions. It should be noted that even after subtraction of shrinkage and thermal strain, the evolution of mechanical strain is affected by humidity and temperature. The reference case is the so-called basic creep, i.e. creep in sealed conditions and at room temperature. Dry concrete creeps less than wet concrete, but the process of drying accelerates transitional creep. Elevated temperatures lead to faster cement hydration and thus to faster reduction of compliance due to aging, but they also accelerate the viscous processes that are at the origin of creep and the process of microprestress relaxation.

According to the solidification theory [3,4], aging of concrete is caused by the formation of new solidified hydration products, mainly calcium–silicate–hydrate gels (C–S–H). The C–S–H is considered as a non-aging constituent with material properties invariable in time, and the evolution of compliance with age is attributed to the growth of the volume fraction (and density) of hydration products. These assumptions lead to the general relation

$$\dot{J}_{ve}(t,t') = \frac{\dot{\Phi}(t-t')}{v(t)} \tag{1}$$

linking the viscoelastic creep compliance rate $\dot{J}_{ve}$ to the rate $\dot{\Phi}$ of the compliance function of the non-aging constituent and to function v describing the growth and densification of hydration products. Subscript ve in J_{ve} emphasizes that this is only a part of the compliance function, caused by viscoelastic effects. Substituting specific expressions for functions Φ and v proposed in [1] based on fitting of experimental data, integrating and adding a term that represents long-term viscous flow, we obtain the basic compliance function of the B3 model in the form

$$J_b(t,t') = q_1 + q_2 \int_{t'}^{t} \frac{ns^{-m}}{s-t'+(s-t')^{1-n}}\,\mathrm{d}s + q_3 \ln\left[1+(t-t')^n\right] + q_4 \ln \frac{t}{t'}. \tag{2}$$

Here, times t and t' are expressed in days, $n = 0.1$ and $m = 0.5$ are fixed exponents, and q_1, q_2, q_3 and q_4 are adjustable parameters. The B3 model provides empirical formulae for estimation of these parameters from concrete composition and strength. The first (constant) term in Eq. (2) corresponds to the compliance $q_1 = 1/E_0$ of the elastic spring in Fig. 1, the second and third terms to the solidifying viscoelastic material (in numerical simulations approximated by a solidifying Kelvin chain), and the fourth term to a viscous dashpot with age-dependent viscosity

$$\eta(t) = \frac{t}{q_4}. \tag{3}$$

The rate of shrinkage strain ε_{sh} at the material point level is assumed to be proportional to the rate of relative humidity h. The proportionality factor denoted as k_{sh} is considered to be a humidity-independent constant.

The effects of temperature and humidity on processes in the microstructure can be described by introducing two transformed time variables: the equivalent age t_e (equivalent hydration period, or "maturity"), which indirectly characterizes the degree of hydration, and the reduced time t_r, characterizing the changes in the rate of bond breakages and restorations on the microstructural level. Under standard conditions, i.e., at room temperature and for sealed specimens, both of these times are by definition equal to the actual age of concrete t. Under higher temperatures, all processes are accelerated, which is taken into account by the acceleration of the transformed times with respect to the physical time. Under lower humidity, all processes are slowed down.

Under general temperature and humidity histories, equations describing the evolution of the microstructure are written in terms of the transformed times, which can be computed by integrating the incremental relations

$$\mathrm{d}t_e = \psi_e(T,h)\mathrm{d}t \tag{4}$$

$$\mathrm{d}t_r = \psi_r(T,h)\mathrm{d}t \tag{5}$$