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A generalized life evaluation formula for uniaxial and multiaxial static fatigue

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Abstract

A generalized life evaluation formula has been deduced based on damage evolution. From the comparison with the experiments in literature, it is found that the formula can well estimate fatigue life for any applied stress level. It is also found that the large scattering of experimental results can be well explained by the random initial damage, and the formula can give the mathematical expect. By introducing the concept of energy-equivalent stress, uniaxial and multiaxial static fatigue problems can be evaluated in a unified way, with the use of S–t curve obtained by uniaxial tests only.

Keywords: Static fatigue; Damage; Evolution law; Life; S–t curve

1. Introduction

Some engineering materials, such as ceramics and glass fibers, etc., may be fractured by fixed loadings below their static strength after certain acting times. Such a delayed fracture is usually called as 'static fatigue'. Many studies [1–5] have been reported on the static fatigue of structural ceramics and concretes, and some mechanisms [6–8] have also been proposed to explain the phenomenon. According to the deformation at fracture, static fatigue can be distinguished into two types, that is, with and without creep/plastic deformation. For the case that there is no creep/plastic deformation, "slow crack growth (SCG)" theory [9,10] has been proposed as the mechanism of static fatigue, and a life evaluation formula $\sigma^m t_f = C$ has been derived from this mechanism. For the case that there is obvious creep/plastic deformation, creep/plastic deformation mechanism has been used to explain the static fatigue, and an empirical life formula $\sigma^m t_f = C$ has been concluded too [11–13]. However, experiments on static fatigue usually are carried out under uniaxial stress states, while a structural material works often under complicate stress state. Moreover, uniaxial experimental results show that stress-life relationship is a curve in a bi-logarithmic diagram (called as S–t curve), and a linear part appears for limited stress levels only. Large scattering of static fatigue tests is unavoidable [14–17]. This particular behavior makes concluding empirical formula from test data difficult. Therefore, from the view of application, a generalized fatigue life evaluation formula is strongly expected.

2. Static fatigue damage evolution law

Damage mechanics [18] have been proved very powerful in fatigue study. The well-known Kachanov's static damage evolution law [19] can be expressed as

$$\frac{dD}{dt} = c\left(\frac{\sigma}{1-D}\right)^{\xi} \tag{1}$$

However, such a damage evolution law can only lead to the life formula $\sigma^m t_f = C$ too. Here, σ is basically a uniaxial stress. For complicate stress state, either von-Mises stress or principal

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stress, may play the equivalent role of uniaxial stress, but there is no experimental evidence yet. From the view of damage mechanics, establishing a reasonable damage evolution law is the key to evaluate fatigue life. Theoretically, a correct damage evolution law should be deduced from the dominate mechanism of static fatigue. However, the mechanism is still argumentative, and it is almost impossible yet to establish a complete damage evolution law (which should be able to describe whole S–t curve, not only the linear part) from current proposed mechanism. So here we consider a macroscopic way to establish the damage evolution law.

A generalized damage evolution law includes two aspects, that is, evaluation parameter and evolution function. For uniaxial stress, obviously the evaluation parameter is just the stress. But what is the parameter under complicate stress state? Damage accumulation process is not an instantaneous fracture, so its dominate parameter may be different from that of fracture. When the damage is accumulated to a critical state under which the effective stress satisfies instantaneous fracture condition, fatigue fracture occurs.

Regarding damage accumulation as an energy dissipation process, strain energy density may be a considerable dominate parameter. Then, the general form of damage evolution can be expressed as

$$\frac{dD}{dt} = f(W, D) \tag{2}$$

where strain energy density W can be expressed in a general form

$$W = \frac{1}{2E}\left[I_1^2 - 2(1+\nu)I_2\right] I_1 = \sigma_1 + \sigma_2 + \sigma_3,$$
$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3 \tag{3}$$

Considering a strain energy equivalent uniaxial problem, here we introduce an effective stress as

$$\sigma_{eq} = \sqrt{I_1^2 - 2(1+\nu)I_2} \tag{4}$$

This effective stress will be called as energy-equivalent stress, and will be used as the dominate parameter under multiaxial stress state below. The damage evolution law can now be proposed by extending Kachnov's law to full S–t curve as

$$\frac{dD}{dt} = c\left(\frac{\sigma_{eq}}{1-D}\right)^{\xi} H \tag{5}$$

Here H is a new term [20] introduced as

$$H = \begin{cases} 1 - \left(\frac{(1-D)\sigma_{f0}}{\sigma_{eq}}\right)^{\zeta} & \text{for } \frac{\sigma_{eq}}{1-D} > \sigma_{f0} \\ 0 & \text{for } \frac{\sigma_{eq}}{1-D} \le \sigma_{f0} \end{cases} \tag{6}$$

where c is the proportional coefficient of damage evolution, ξ, ζ are material constants, and σ_{f0} is the nominal fatigue limit of material (a material constant), which is a new concept introduced to express the variation of actual fatigue limit with damage state. The actual fatigue limit σ_f of material depends on its current damage value D as

$$\sigma_f = (1-D)\sigma_{f0} \tag{7}$$

Denoting the initial damage as D_0, then the original actual fatigue limit is $\sigma_f = (1-D_0)\sigma_{f0}$. Therefore, the actual static fatigue limit may degrade if damages have been accumulated. The physical meaning of nominal fatigue limit can be simply imaged as the actual fatigue limit corresponding to very small initial damage $D_0 \approx 0$. However, in materials that static fatigue occurs, for examples, in sintered ceramics with porosity, there usually would be relatively large initial damage already. So attention is needed to pay that the measured fatigue limit corresponding to initial damage D_0 is not the nominal fatigue limit.

3. Generalized static fatigue life formula and S–t curve

Integrating Eq. (5), one gets the complete mathematical expression of S–t curve as

$$\sigma_{eq}^{\xi} t_f = C\int_{D_0}^{D_C} \frac{(1-D)^{\xi}}{H} dD = CI(\sigma_{eq}) \tag{8}$$

$$I(\sigma_{eq}) = \int_{D_0}^{D_C} \frac{(1-D)^{\xi}}{H} dD \tag{9}$$

where D_C is the critical damage (but it is not a fixed value, and will be determined later) at which failure happens, and D_0 is the initial damage. $C = 1/c$ is the proportional coefficient of life, with a dimension of $(MPa)^{\xi} \cdot s$ (but it will be omitted for the simplicity below). Comparing Eq. (8) with the widely used empirical formula $\sigma^m t_f = C$, it can be seen that the difference is only due to the additional non-dimension term $I(\sigma)$. This term enable us to express the whole of S–t curve. The integral of Eq. (9) can be calculated by numerical integral procedure. Introducing the transformation

$$D = \frac{D_C - D_0}{2}\eta + \frac{D_C + D_0}{2} \tag{10}$$

one gets the standardized numerical integration formula

$$I(\sigma_{eq}) = \frac{D_C - D_0}{2}\int_{-1}^{1} \frac{(1-D)^{\xi}}{H} d\eta = \frac{D_C - D_0}{2}\sum_{i=1}^{M} \frac{(1-D_i)^{\xi}}{H_i} w_i \tag{11}$$

where, M is the number of integral points, η_i, w_i are the integral point and weight coefficient, respectively. And

$$D_i = \frac{D_C - D_0}{2}\eta_i + \frac{D_C + D_0}{2} \tag{12a}$$

$$H_i = \begin{cases} 1 - \left(\frac{(1-D_i)\sigma_{f0}}{\sigma_{eq}}\right)^{\varsigma} & \text{for } \frac{\sigma_{eq}}{1-D_i} > \sigma_{f0} \\ 0 & \text{for } \frac{\sigma_{eq}}{1-D_i} \le \sigma_{f0} \end{cases} \tag{12b}$$

The critical damage D_C, at which instantaneous fracture occurs, is not a fixed value, but should be determined by the fracture condition as

$$\sigma_{ef} = \frac{\sigma_1}{1-D_C} = \sigma_{b0} \tag{13}$$

where σ_1 is the first principal stress, σ_{b0} is the nominal tensile strength of material corresponding to initial damage $D_0 \approx 0$.